Operations Scheduling by Markov Chains with Strong and Weak Interactions, by D. Teneketzis, H. Javid and B. Sridhar, Systems Control, Inc., 1801 Page Mill Road, Palo Alto, CA 94304

Abstract. An example from hydroscheduling motivates the study of Markov chains with strong and weak interactions. A control scheme is developed which takes advantage of the weak coupling and provides a suboptimal solution which differs from the optimal by not more than the order of the strength of the coupling.

1. Introduction. Operations scheduling in electric power systems involves decisions made over time-scales from hours (short term hydroscheduling, unit commitment) to years (long-term hydro reserve management, nuclear refueling maintenance scheduling). This problem can in principle be solved by Stochastic Dynamic Programming. However, the computational intractability of this approach has led practitioners to partition the operations scheduling problem along heuristic lines, resulting in a set of subproblems treated as though each were independent of the others. Since operations scheduling comprises short term, midterm and long term problems, time scale decomposition can be used in order to develop rigorous procedures for separating the overall problem into subproblems, and for recomposing subproblem solutions into an overall solution.

Operations scheduling under uncertainty is a constrained stochastic optimization problem. One way of solving constrained stochastic optimization problems is through stochastic approximations [1]. Stochastic approximations lead to controlled Markov chain problems for which algorithms have been developed [2]. The presence of short term (fast) and long term (slow) problems in operations scheduling motivates the study of Markov chains with strong and weak interactions.

In this paper we develop a decomposition algorithm for suboptimal control of Markov chains with strong and weak interactions. The paper is organized as follows: In Section 2 an example from hydroscheduling is presented where the existence of time scales is revealed. A constrained stochastic control problem is formulated and stochastic approximations are used to yield a control problem for Markov chains with strong and weak interactions. In Section 3 an algorithm for suboptimal control of Markov chains with strong and weak interactions is presented. A summary is presented in Section 4.

2. An Example from Hydro Scheduling. As a starting point we concentrate on the optimization of hydro resources.

2.1 The Model. The dynamic equations for the reservoirs can be written in the form

(1) $$dx' = (w' - u' - s' + B'u' + C's')d\tau$$

where τ is in units of weeks and

- x' = state vector with components $x_i'(\tau)$, $i=1,\ldots,N$ which are the amount of water stored in reservoir i at time τ.
- u' = decision vector with components $u_i'(\tau)$, $i=1,\ldots,N$ which are the rates of water release from reservoir i for electric generation at time τ.
- s' = decision vector with components $s_i'(\tau)$, $i=1,\ldots,N$ which are the rates of water spillage at time τ.
- B' = NxN water transfer matrix, where the component b_{ij}' denotes the fraction of the rate of release u_j' which becomes available to reservoir i (typically, $b_{ij}' = 0$ or 1).
- C' = NxN water transfer matrix, where the component c_{ij}' denotes the fraction of the rate of spillage v_j' which become available to reservoir i (typically, $c_{ij}' = 0$ or 1).
- w' = inflow vector with components $w_i'(\tau)$, $i=1,\ldots,N$, denoting the rate of inflow into reservoir i at τ.

Note that this model neglects delays between reservoirs; these can be accommodated by appropriate modifications to (1).

The release rates for electric generation and spillage are constrained by

(2) $0 \leq u'_i \leq u_{imax}$

(3) $0 \leq s'_i \leq s_{imax} \qquad i=1,\ldots,N.$

The maximum spillage constraint is for flood control, fish-habitat maintenance, adherence to water laws, etc.; this constraint is not normally active. The states are constrained by

(4) $0 \leq x'_i \leq x_{imax} \qquad i=1,2,\ldots,N$

A number of inflow models are possible. Here we assume that the reservoir inflows take the form

(5) $dw'_i = f_i(\tau, w'_i(\tau))d\tau + \sqrt{g_i(\tau, w'_i(\tau))}\, dB^i_\tau \qquad i=1,\ldots,N$

where f_i and g_i may be obtained by identification [5] and the B^i_τ are independent Brownian motions.

The load may be modeled as the diffusion process [5]

(6) $dD = \varepsilon\mathscr{B}(\varepsilon\tau,\tau,D)d\tau + \sqrt{\alpha\varepsilon}\, dB_\tau$

where D has periods, dependent on $\mathscr{B}$, in $\varepsilon\tau$(years) and τ (weeks) where $\varepsilon = 1/52$ is a time scaling parameter. The load may also be modeled as a Markov chain as described in [6].

Scaling of Variables to Reveal Time Scales

Defining

$$(7) \quad \begin{cases} x_i = x'_i/x_{imax} \\ u_i = u'_i/u_{imax} \\ s_i = s'_i/u_{imax} \\ w_i = w'_i/u_{imax} & i=1,2,\ldots N \\ b_{ij} = b'_{ij}\dfrac{u_{jmax}}{u_{imax}} \\ C_{ij} = C'_{ij}\dfrac{u_{jmax}}{u_{imax}} \end{cases}$$

$$(8) \quad H' = \text{diag}\left[\frac{u_{1max}}{x_{1max}}\ \frac{u_{2max}}{x_{2max}} \cdots \frac{u_{Nmax}}{x_{Nmax}}\right]$$

we obtain the transformed reservoir dynamics and the inflows equations

(9) $dx = H'(w-u-s + Bu + Cs)d\tau$

(10) $$dw_i = f_i(\tau_i, w_i(\tau))d\tau + \sqrt{g_i'(\tau, w_i(\tau)}\; dB_\tau^i$$

Assume that the components of x in (1) or x in (9) are ordered so that

(11) $$x_{1max} \geq x_{2max} \geq \cdots \geq x_{Nmax},$$

the first N_S reservoirs have sufficient capacity to be considered seasonal or annual reservoirs, and the last $N_F = N - N_S$ reservoirs are weekly reservoirs, with k defined such that

(12) $$x_{(N_S+1)max} = \varepsilon k\, X_{N_S max}$$

where k is a constant and $k\varepsilon \ll 1$. Equation (9) may be written

(13) $$dx = \begin{bmatrix} \varepsilon K I_{N_S} & 0 \\ 0 & I_{N_F} \end{bmatrix} H(w-u-s + Bu + Cs)d\tau.$$

I_{N_S} and I_{N_F} are identity matrices of appropriate order, and

(14) $$H = \text{diag}\left[\alpha \frac{u_{1max}}{x_{1max}} \cdots \alpha \frac{u_{N_S max}}{x_{N_S max}} \frac{u_{(N_S+1)max}}{x_{(N_S+1)max}} \cdots \frac{u_{Nmax}}{x_{Nmax}}\right]$$

(15) $$\alpha = \frac{x_{N_s max}}{x_{(N_S+1)max}}$$

where the diagonal elements of H are of the same order. Equation (13) shows that the first N_s reservoirs change levels much slower than the levels of the last N_F reservoirs.

<u>Cost Function</u>

For the hydro scheduling problem formulation treated here, the value of water is the cost of the displaced thermal generation; the associated optimization problem is to minimize production cost. Let $e_i(x_i, u_i)$ be the amount of electricity generated by release rate u_i when the amount of water in reservoir i is x_i. Then given that the demand is D, the thermal part of demand the power system must generate

(16) $$T = D - \sum_{i=1}^{N} e_i\,(x_i, u_i)$$

and the production cost is

(17) $$c(T) = c(D - \sum_{i=1}^{N} e_i\ (x_i,\ u_i))$$

where $c(\cdot)$ is usually a convex function of the required thermal generation T.

2.2 The Optimization Problem. The optimization problem can be stated as follows:

$$\left.\begin{array}{ll} & \underset{\{u_1^*,\ s_i^* | i=1,\dots N\}\ t\varepsilon[0,T]}{\text{Minimize}}\ J(u,s) \\ & \text{subject to (6), (12), (17) and} \\ (18) & 0 \le x_i \le 1 \\ (19) & 0 \le u_i \le 1 \\ (20) & 0 \le s_i \le \dfrac{s_{imax}}{u_{imax}} \end{array}\right\}(p)$$

where

(21) $$J(u,s) \triangleq \lim_{T\to\infty} \frac{1}{T}\ E\left\{\int_o^T c(D - \sum_{i=1}^{N} e_i(x_i,u_i))d\tau\right\}.$$

Sufficient Conditions for Optimality

The sufficient conditions for optimality for problem (p) are given by the Hamilton-Jacobi equation (HJE):

$$\frac{\partial J^\varepsilon}{\partial \tau} + \varepsilon\mathscr{B}(\varepsilon\tau,\tau,D)\ \frac{\partial J^\varepsilon}{\partial D} + \frac{1}{2}\ \varepsilon\alpha\ \frac{\partial^2 J^\varepsilon}{\partial D^2} +$$

$$+ \underset{u,s}{\text{Min}} \left[\sum_{i=1}^{N_S} \alpha\ \frac{u_{imax}}{x_{imax}}\ [w_i - u_i - s_i + \sum_{j=1}^{N} (b_{ij}u_j + c_{ij}s_j)]\ \frac{\partial J^\varepsilon}{\partial x_i}\ \varepsilon k\right.$$

$$+ \sum_{i=N_S+1}^{N} \frac{u_{imax}}{x_{imax}}\ [w_i - u_i - s_i + \sum_{j=1}^{N} (b_{ij}u_j + c_{ij}s_j)]\ \frac{\partial J^\varepsilon}{\partial x_i}$$

$$+ \sum_{i=1}^{N} [f_i(\tau,\ u_{imax}\ w_i(\tau))\ \frac{\partial J^\varepsilon}{\partial w_i} + \frac{1}{2}\ g_i(\tau,\ u_{imax}\ w_i(\tau))\ \frac{\partial^2 J^\varepsilon}{\partial w_i^2}\ ]$$

$$+ c \left(D - \sum_{i=1}^{N} e_i(x_i,u_i)\right)\Bigg] = 0$$

where J^{ε} is the optimal cost.

An approximate solution to the optimization problem may be obtained by discretizing the HJE by the method of stochastic approximations to obtain an optimization problem for Markov chains [1]. Discretization results in the following optimization problem.

(23) $$\underset{\{u_i,s_i \mid i=1,\ldots,N\}}{\text{Minimize}} \; J_{\varepsilon}(u,s) = \frac{1}{1+T} E\sum_{\tau=0}^{T} \overline{c}\left(D - \sum_{i=1}^{N} e_i(x_i,u_i)\right)\Delta\tau$$

subject to

(24) $$u(i) \in U(i) \quad \forall_i$$

(25) $$s(i) \in S(i), \ \forall_i$$

(26) $$p(\tau+\Delta\tau) = p(\tau)\ [A(u,s) + \varepsilon B(u,s]$$

(R)

where U(i), S(i) are prespecified sets, the ℓ-dimensional row vector $p(\tau)$ has elements $p_n(\tau)$ which represent the probability that state n is occupied at time τ, the matrix A(u,s) is block diagonal,

(27) $$A(u,s) = \text{diag}[A_{11}(u,s)\ A_{22}(u,s)\ldots A_{kk}(u,s)],$$

and the elements $a_{ij}(u,s)$ and $b_{ij}(u,s)$, $i,j=1,\ldots,\ell$, satisfy

(28) $$b_{ij}(u,s) \geq 0 \text{ if } a_{ij}(u,s) = 0$$

(29) $$\sum_{j=1}^{\ell} b_{ij}(u,s) = 0$$

This chain is referred to as a Markov chain with strong and weak interactions or as a weakly coupled Markov chain. The strength of the coupling is dependent on the small parameter ε.

3. Control of Markov Chains with Strong and Weak Interactions. In order to develop a decomposition algorithm that provides a near optimal solution to problem (R) we need to introduce first the aggregate Markov chain associated with that of (26).

The Aggregate Markov Chain

The aggregate Markov chain corresponding to the weakly coupled Markov chain described by (34) is defined by

$$\Pi(t+1) = \Pi(t)Q(u,s) \tag{30}$$

where

$$Q(u,s) = I + \varepsilon M(u,s) \tag{31}$$

and the elements $m_{ij}(u)$ of $M(u)$ satisfy *

$$m_{ij}(u,s) = v_i(u,s)\ B_{ij}(u,s)\ \underline{1} \tag{32}$$

$$\underline{1} = [1,\ldots,1]' \tag{33}$$

and $v_i(u,s)$ is the steady state probability distribution of the chain

$$v_i(t+1) = v_i(t)\ A_{ii}(u,s)\ . \tag{34}$$

The number of states in the aggregate chain is equal to the number of groups of weakly coupled states in the original chain.

The following assumptions are made in order to study the control problem (R):

(A1) For all $(u(i),s(i))\varepsilon U(i)xS(i)$ $i=1,2,\ldots\ell$ the weakly coupled Markov chain has a single ergodic class [2].

(A2) For each group I_i of the weakly coupled states and for all $(u(j),s(j))\varepsilon U(j)xS(j)$ the matrices $A_{ii}(u,s)$ are regular [3].

(A3) For all $(u(i), s(i))\varepsilon U(i)xS(i)$ the aggregate chain has a single ergodic class.

(A4) The instantaneous cost $c(D - \sum_i e_i(x_i,u_i))$ is uniformly bounded for $x_i\varepsilon\{1,2,\ldots\ell\}$ and $(u(j),s(j))\varepsilon U(j)xS(j)$.

(A5) For each $i,j\varepsilon\{1,2,\ldots\ell\}$ the functions

$$\text{Prob}\{x_{t+1} = j|x_t = i\} = a_{ij}(u,s) + \varepsilon\ b_{ij}(u,s)$$

as well as the instantaneous cost are continuous with respect to $(u(i),s(i))$.

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*$B(u,s)$ is partitioned in a way compatible with $A(u,s)$.

(A6) The sets $U(i), S(i)$ of admissible controls for each state i are compact.

Under the above assumptions the following results have been obtained [4]:

Theorem 1 [4]

Under (A1) - (A3), for any policy $(u(i),s(i))\varepsilon U(i)xS(i)$ $i=1,2,\ldots\ell$, the components $p_i(u,s)$ of the steady-state probability distribution of the weakly coupled Markov chain, $p(u,s)$, satisfy

(35) $$p_i(u,s) = (\Pi_j(u,s) + 0(\varepsilon))(v_i(u,s) + 0(\varepsilon)) \quad i\varepsilon I_j, j=1,2,\ldots K$$

where $\Pi_j(u,s)$ is the steady-state probability distribution of the aggregate chain. ■

Theorem 2 [4]

Under assumptions (A1) - (A6) the cost $J_\varepsilon(u)$ corresponding to any fixed policy $(u(i),s(i)\varepsilon U(i)xS(i)$, $i=1,2,\ldots\ell$ can be approximated by

(36) $$J_\varepsilon(u,s) = J_o(u,s) + 0(\varepsilon)$$

where

(37) $$J_o(u,s) = \sum_{j=1}^{k} \Pi_j(u,s) \sum_{i\varepsilon I_j} v_j^i(u,s)\ \bar{c}(D,x_i,u_i)$$

and

(38) $$\bar{c}(D,x_i,u_i) = \bar{c}(D - \sum_{i=1}^{N} e_i(x_i,u_i)). \blacksquare$$

Theorem 3 [4]

Let assumptions (A1) - (A6) be satisfied, (u^o,s^o) be the policy minimizing $J^o(u,s)$, i.e.

(39) $$J_o(u^o,s^o) = \min_{(u,s)\varepsilon UxS} J_o(u,s),$$

$$(UxS=U(1)x\ldots U(\ell)xS(1)x\ldots xS(\ell)),$$

and $(u^*(\varepsilon),s^*(\varepsilon))$ be the policy minimizing $J_\varepsilon(u)$, i.e.

(40) $$J_\varepsilon\ (u^*(\varepsilon),s^*(\varepsilon)) = \min_{(u,s)\varepsilon UxS} J_\varepsilon(u,s)$$

Then,

(41) $$\lim_{\varepsilon\to 0} (J_o(u^o,s^o) - J_\varepsilon(u^*(\varepsilon),s^*(\varepsilon))) = 0 \quad \blacksquare$$

The above results suggest the following two step algorithm that computes a near optimal solution to problem (R):

First Step

Let $\varepsilon=0$ solve for $v_j(u,s)$ from (34) $(j=1,2,\ldots k)$ and compute the costs

$$\mathscr{L}_j = \sum_{i\varepsilon I_j} v_j^i(u(i),s(i))\ \bar{c}(D,x_i,u(i)). \tag{42}$$

Second Step

Compute $Q(u,s)$ using $v_j(u,s)$ from Step 1 and (32), (33). Then compute $\Pi_j(u,s)$ and $J_o(u,s)$ from (30) and (37) using the costs $\mathscr{L}_j$ from Step 1.

In terms of the hydroscheduling problem this algorithm suggests the following: Instead of solving the overall hydro scheduling problem at once, solve first the short term problem; imbed the solution of the short term problem into the midterm problem and solve the midterm problem; imbed the solution of the midterm problem into the long term problem and solve the long term problem. Each of the above problems is of smaller dimension than the original overall problem and the performance achieved is nearly optimal. Under certain conditions we can proceed further and reduce the computational requirements for the control of the aggregate chain by decomposing the policy space as indicated below.

Assume that

(A7) UxS can be partitioned into sets $R_1\ R_2\ldots R_M$ such that

(43) (i) $UxS = \bigcup_{i=1}^{M} R_i$

(44) (ii) $R_i \cap R_j = \phi \quad \forall(i,j) \qquad i\neq j$

(iii) $\forall(u,s)\varepsilon R_i$ the matrix of transition probabilities $Q(u,s)$ for the aggregate Markov chain is the same, i.e.

$$Q(u_i,s_i) = Q(u_r,s_r)\ \forall((u_i,s_i),\ (u_r,s_r))\varepsilon R_i \tag{45}$$

Then we have the following result:

Theorem 4 [4]

Assume (A1) - (A7). Consider (u_m,s_m), $(u_r,s_r)\varepsilon R_s$ such that

(46) $$\sum_{i=1}^{n_i} v_j^i(u_m,s_m)\ \bar{c}(D,x_i,u_m(i))$$

$$\leq \sum_{i=1}^{n_i} v_j^i(u_r,s_r)\ \bar{c}(D,x_i,u_r(i))\ \forall I_j$$

Then (u_r,s_r) does not need to be considered in the control of the aggregate Markov chain. ■

Theorem 4 shows that among all the policies that result in the same long run behavior of the system (i.e. the same Q(u,s)), only those which result in the minimum short run cost (i.e. minimum costs for the fast Markov chain control problem) should be considered for the long run optimization problem (i.e. the control problem for the aggregate chain).

A simple example presented in [4] illustrates the decomposition algorithm as well as the decomposition in policy space.

4. Summary. An example from hydro scheduling motivated the study of Markov chains with strong and weak interactions. A control algorithm which takes advantage of the weak coupling and provides a sub-optimal solution differing from the optimal by not more than the order of the strength of the weak interaction was developed. Finally a decomposition of the policy space which reduces the computational requirements for the control of the aggregate Markov chain was presented.

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