A Surrogate Optimization-Based Mechanism for Resource Allocation and Routing in Networks With Strategic Agents

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***Abstract*—We consider a mechanism design problem for the joint flow control and multipath routing in informationally decentralized networks with strategic agents. Based on a surrogate optimization approach, we propose an incentive mechanism that strongly implements the social-welfare maximizing outcome in Nash equilibria. This mechanism possesses several other desirable properties, including individual rationality and budget balance at equilibrium. More importantly, in contrast to the existing literature on the network resource allocation mechanisms, the proposed mechanism is dynamically stable, meaning that the Nash equilibrium (NE) of the game induced by the mechanism can be learned by the agents in a decentralized manner. To establish dynamic stability, we propose a decentralized iterative process that always converges to a NE of the game induced by the mechanism, provided that all strategic agents follow the process. To the best of our knowledge, this is the first incentive mechanism that simultaneously possesses all the above-mentioned properties.**

***Index Terms*—Joint flow control and multipath routing, mechanism design, network resource allocation, strategic agents.**

I. Introduction

RESOURCE allocation is an essential part of the management of communication networks. The purpose of resource allocation is to intelligently assign the limited available network resources among the network's users/agents so as to optimize network performance.

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The design of resource allocation mechanisms becomes very challenging when the information needed to make efficient allocations is distributed among the network's agents who are strategic, that is, each network agent possesses private information and her objective is to maximize her own utility. In these instances, due to the (partial) conflict among agents' objectives and the network-wide performance objective, agents do not voluntarily reveal their private information to the network manager, who is responsible for allocating the network's resources. Therefore, to elicit the information the network manager needs from the agents, he must provide external incentives to them so as to align their individual objectives with the network-wide performance objective. The design of such incentives must address the following challenges: It must be based on the information revealed by the network agents. It must anticipate the agent's strategic behavior in their revelation of information according to the created incentives.

One way to address these challenges is to design incentive resource allocation mechanisms that consist of two components: 1) a message/strategy space, that is, a communication alphabet through which the agents can send information to the network manager; and 2) an outcome function that determines the allocation and monetary incentives (payments) after the communication and information exchange process terminates. Each resource allocation mechanism together with the agents' utilities induce a game among the agents. The interaction of the strategic agents in this game determines the allocation of the resources in the network.

In this paper, we study a model of network resource allocation where the network agents are strategic and possess private information. The problem is motivated by joint flow control and multipath routing. Specifically, we consider a communication network with arbitrary topology, a set of strategic agents who wish to use the network's resources, and a network manager who knows the network's topology and resources and has to allocate these resources among the strategic agents. Each strategic agent wishes to transmit data from a source to a destination; transmission may take place via multiple paths. Each agent's satisfaction relates to her transmission rates over her multiple paths through a valuation function. Each agent's valuation function is her own private information. The network manager's objective is to allocate the network's resources among the strategic agents so as to maximize the social-welfare, i.e., the sum of agents' valuation functions. Therefore, the network manager is faced with an optimization problem where the information about the objective function is distributed among agents who are only working toward maximizing their own utility. Thus, in order to incentivize strategic agents to reveal the information needed for maximizing the social welfare, the network manager has to provide external incentives to them to align their individual objectives with the social objective.

As pointed out above, one approach the network manager can pursue to achieve his goal is to design an incentive resource allocation mechanism consisting of a message space and an outcome function that determines rate allocations and monetary incentives. One of the challenges associated with the determination of an incentive mechanism that leads to the social-welfare maximizing outcome is due to the fact that, in our problem, rate allocations across different links of a route are *coupled*, as the rate an agent receives along every link of a route must be the same. Therefore, the network manager needs to design a *coupled* resource allocation mechanism to achieve his desirable outcome.

The coupled resource allocation mechanism designed by the network manager must take into account the problem's information structure (who knows what), the agents' strategic behavior, and must possess certain desirable properties.

- (P1) It must ensure the existence of at least one equilibrium in every game induced by the mechanism.
- (P2) It must strongly implement[1] the social-welfare maximizing solution in every equilibrium of the induced game.
- (P3) It must balance the budget at equilibrium, i.e., it must ensure that at equilibrium the sum of monetary incentives (taxes or subsidies) provided to all strategic agents is equal to zero;
- (P4) it must ensure that the strategic agents voluntarily participate in the resource allocation mechanism; and
- (P5) it must be dynamically stable, i.e., there must be a tatonnement/iterative process along which the equilibrium can be learned/attained.

The meaning of properties (P1)–(P5) will be fully explained in Section III of the paper. The most preferable equilibrium concept for problems with private information is dominant strategy equilibrium; however, the only mechanism that possesses property (P2) in dominant strategy equilibrium is the Vickery–Clarke–Groves (VCG) mechanism [2]–[4], which is not budget balanced. Therefore, research efforts aimed at designing resource allocation mechanisms have generally focused on achieving efficiency in *Nash equilibrium* (NE) [5].

NE is a solution concept that is usually applied to games with complete information; thus, some justification is needed for applying it to a game with private information. A plausible justification is the alternative interpretation of NE provided originally by Nash [6], and later adopted by Reichelstein and Reiter in [7], Groves and Ledyard in [8] and many others. According to this interpretation, the complete information NE may be the possible equilibrium of an iterative learning process, that is, the stationary message profile of the iterative process. Therefore, NE can be considered as a practical solution concept for the game induced by the mechanism, if and only if, the network manager specifies an iterative process through which the agents can find the NE of the induced game. The above-mentioned discussion demonstrates the importance of property (P5), which is not provided in most of the existing literature (see [9]–[16]).

In this paper, considering NE as the solution concept, we present a resource allocation mechanism for the joint flow control and multipath routing problem that possesses properties (P1)–(P5). The mechanism is based on the *surrogate optimization approach* to mechanism design. In this approach, the network manager uses the agents' messages to approximate each agent's valuation function by a surrogate function, and then, optimizes the sum of surrogate functions under the network constraints. Surrogate optimization was first used by Yang and Hajek in [9] to solve the flow control problem in networks; the same approach was subsequently used in [10]. In [9] and [10], the agents' payments are based on the charging policy of the VCG mechanism. Thus, the mechanisms proposed in [9] and [10] are not budget balanced; furthermore, they do not achieve strong implementation in NE and do not address dynamic stability. Our approach presented in Section IV overcomes the drawbacks of the mechanisms of [9] and [10] and comes up with a mechanism that simultaneously achieves Nash existence, strong implementation, budget balance (BB), individual rationality (IR), and dynamic stability [properties (P1)–(P5)]. We satisfy dynamic stability by proposing a decentralized iterative process, which converges to a NE and requires that agents update their messages based only on *local* information.

In addition to the main properties (P1)–(P5), our proposed mechanism has the following additional features that facilitate its implementation.

- (S1) *Uniqueness of the NE:* The NE of the game induced by the mechanism is typically unique, that is, in most of the cases, the game induced by the mechanism has a unique NE. Uniqueness of NE is a desirable feature as it eliminates the issue of equilibrium selection (in games with multiple equilibria, it is unclear how players are expected to coordinate when each of the agents might prefer one of the equilibria).
- (S2) *Feasibility out of equilibrium:* At all outcomes of the game induced by the mechanism, both at and off equilibrium, the allocation assigned to the agents is feasible, i.e., satisfies all the network constraints. Feasibility out of equilibrium is vital when discussing dynamic stability, as it ensures that allocations are still feasible when play has not yet converged.

A. Review of Related Literature

There is a growing body of literature on resource allocation for informationally decentralized networks. Most of the existing literature (e.g., [17]–[23]) has approached the design of decen-

[1]We use the notion of "strong implementation" as introduced in [1] to ensure that the efficient allocation is implemented by *all* equilibria of the induced game, not just *some* of them. Achieving strong implementation is clearly more difficult than achieving truthful or partial implementation that requires only that some (but not all) of the equilibria induce efficient allocation.

tralized resource allocation mechanisms under the assumption that the agents are cooperative (non-strategic), that is, they obey the rules of the proposed decentralized mechanism.

In the literature on decentralized resource allocation for networks with strategic agents, most of the works have focused on the flow control problem. In [24] and [25], the authors study the distributed flow control mechanism of Kelly *et al.* [18] in networks with strategic agents and show that in the presence of strategic behavior, the allocations corresponding to the Nash equilibria are different from the efficient allocation. The flow control problem with strategic agents has also been addressed by VCG-based mechanisms that appear in [9]–[12], [26], and [27]. These mechanisms are not budget balanced and in most of them [9]–[12] the allocations corresponding to the NE of the induced game are not always socially efficient, that is, these mechanisms do not strongly implement the optimal solution of the centralized problem in NE. Furthermore, these works do not address the issue of dynamic stability. The mechanisms presented in [13] and [14] achieve strong implementation, IR and BB, but do not provide a tatonnement process for reaching the NE outcome.

Joint flow control and multipath routing problems have been studied in [15] and [16]. The mechanisms presented in [15] and [16] achieve implementation in NE but do not address dynamic stability, that is, they do not present a tatonnement process that converges to the NE of the game induced by the mechanism. Furthermore, the mechanism presented in [15] is not budget balanced.

The discovery of tatonnement processes/algorithms that converge to a NE of a game has been investigated by many researchers. Several authors, beginning with Milgrom and Roberts [28], have suggested *supermodularity* as a condition sufficient to guarantee the convergence of a class of tatonnement processes to the smallest interval containing all the NE of a game [29]; this kind of convergence is not an appropriate stability concept in a mechanism design setting (see [30]). Many authors, beginning with Monderer and Shapley [31], studied potential games and proved that in such games the best-response (BR) tatonnement process can guarantee convergence to one of the NE of the game [31], [32]. To the best of our knowledge, none of the incentive resource allocation mechanisms proposed so far induce potential games among network agents.

There are few papers that address dynamic stability in mechanism design settings [30], [33]–[36]. The works of Healy and Mathevet in [30], and Yang and Hajek in [33] deal with single-link networks (networks with a single divisible good). For these single-link networks, Yang and Hajek [33] proposed a tatonnement process that converges to the NE point of a class of efficient mechanisms, called g-mechanisms. In [30], using the concept of contraction mappings, the authors designed a mechanism for the allocation of a single divisible good that is stable under the BR tatonnement process. The work of Sinha and Anastasopoulos [34] considers an environment with *multiple independent* goods and applies the mechanism proposed in [30] for each individual good separately. In [35] and [36], the authors consider an environment with multiple coupled goods and propose a tatonnement process that is a version of the Clarke–Groves mechanism; this process achieves efficient allocation but does not achieve BB.

The dynamic stability problem we address in this paper is also related to the literature on NE seeking (see [37] and [38]). The work presented in [37] proposes gradient-based strategies for NE seeking in quadratic or a special class of nonquadratic games. Salehisadaghiani and Pavel in [38] propose gossip-based algorithms to find the NE of the corresponding game. These algorithms work under the assumption that the gradient of the utility functions is Lipschitz continuous in the allocation. This assumption is not satisfied in any incentive resource allocation mechanism proposed so far.

The problem we study in this paper considers strategic agents, thus it is different from the literature on network resource allocation with cooperative agents [17]–[23]. It is different from the works focused on the flow control problem [9]–[14], [24]–[27], as it addresses the joint flow control and routing problem. The allocation mechanism proposed in this paper is distinctly different from all other mechanisms appearing in the literature on network resource allocation with strategic users (see [9]–[16], [26], [27], [30], [33], and [34]) for the following reasons: (i) it deals with networks with arbitrary topology, unlike the mechanisms of [26], [27], [30], [33], [34] that are designed for single-link networks; (ii) it is the only mechanism that possesses properties (P1)–(P5) for networks with multiple links.

The dynamic stability issue we address in this paper is also different from the works appearing in [28], [29], [31], [32], [37], and [38]. One of the goals of this paper is to design a mechanism that induce games in which the NE can be learned; the authors of [28], [29], [31], [32], [37], and [38] consider a given/fixed game with some special properties and provide algorithms that converge to a NE.

B. Contributions of the Paper

We consider a mechanism design problem for implementing the optimal (centralized) solution of the joint flow control and multipath routing problem in networks with an uninformed network manager and strategic agents who possess private information. The problem deals with the allocation of the network's links' bandwidth to the network's strategic agents. Efficient allocations cannot be done independently from link to link (as explained in Section I). Thus, we are faced with the allocation of many *coupled goods* (where a good is the capacity of a network link). Using a surrogate optimization approach, we propose a mechanism for networks with arbitrary topology that simultaneously satisfies several desirable properties, including strong implementation, IR, and BB at equilibrium. Furthermore, we establish dynamic stability of our mechanism by designing a best-estimate (BE) tatonnement/iterative process that always converges to a NE of the game induced by the proposed mechanism when strategic agents update their messages according to it. In our proposed tatonnement process, the update rule of each agent is local, in the sense that the agents do not need to have any information about the valuation functions, actions, or allocations of the other agents. Each agent can update her message based only on her own local information.

To the best of our knowledge, this is the first paper which presents a mechanism that addresses the allocation of multiple coupled goods to strategic agents, and simultaneously satisfies strong implementation in NE, budget balance, individual rationality, and dynamic stability.

C. Organization of the Paper

The rest of the paper is organized as follows. We present our model for joint flow control and multipath routing problem with an uninformed network manager and strategic agents in Section II. In Section III, we formulate the network manager's problem as a mechanism design problem and describe our design goals. We describe the incentive resource allocation mechanism we propose for the solution to this problem in Section IV. We establish the properties of the proposed mechanism at equilibrium in Section V. In Section VI, we establish the dynamic stability of the mechanism by proposing a tatonnemnet process under which the NE can be learned. We conclude our paper in Section VII.

II. Model

Consider a communication network with point-to-point directional links, each having a fixed capacity. Denote the set of links by $\mathcal{L} = \{1, \ldots, L\}$, and the capacity of link $l \in \mathcal{L}$ by c_l. There are N strategic agents in the network, each wishing to transmit data from a source to a destination, and one network manager who allocates the network's resources to the strategic agents. We denote the set of strategic agents by $\mathcal{N} = \{1, \ldots, N\}$. Each agent $i \in \mathcal{N}$ has K_i available paths or routes connecting her source to her destination. Let R_i^k, $k = 1, \ldots, K_i$, denote the set of links that form the kth available route for agent i. We express the transmission rate of data sent over different routes of agent i by the vector $\boldsymbol{x}_i = (x_i^1, \ldots, x_i^{K_i})$, with x_i^k denoting the transmission rate of agent i along route R_i^k.

Each strategic agent $i \in \mathcal{N}$ has a valuation for transmitting data with rates $x_i^1, \ldots, x_i^{K_i}$; this valuation, denoted by $V_i(x_i^1, \ldots, x_i^{K_i})$, expresses the monetary value or satisfaction that agent i derives from transmitting data with the rates $x_i^1, \ldots, x_i^{K_i}$. Agent i's valuation function belongs to a commonly known function space $\mathcal{V}_i$, but its realization $V_i(.) \in \mathcal{V}_i$ is agent i's private information. We make the following assumption about the function spaces $\mathcal{V}_i$, $i \in \mathcal{N}$.

Assumption 1: For each $i \in \mathcal{N}$, each valuation function $V_i(.) \in \mathcal{V}_i$ is continuous, strictly concave, and differentiable; $V_i(x_i^1, \ldots, x_i^{K_i})$ is strictly increasing in each $x_i^k, k = 1, \ldots, K_i$, and $V_i(\mathbf{0}) = 0$, where $\mathbf{0}$ denotes the K_i-dimensional zero vector. We do not assume any specific functional form for the elements of $\mathcal{V}_i$, thus the function space $\mathcal{V}_i$ is infinite-dimensional.

Let t_i denote the monetary payment made by agent i to the network. This payment can be any positive or negative real number; if $t_i > 0$, then agent i pays money (tax), whereas $t_i < 0$ implies that agent i receives money (subsidy). The total utility of agent i is given by

$$U_i(\boldsymbol{x}_i, t_i) = V_i(x_i^1, \ldots, x_i^{K_i}) - t_i. \tag{1}$$

Each strategic agent i makes strategic decisions to maximize her own utility function U_i. However, the network manager's objective is to maximize the network's overall satisfaction to its users, expressed by the social welfare function W defined by

$$W(\boldsymbol{x}) = \sum_{i \in \mathcal{N}} V_i(x_i^1, \ldots, x_i^{K_i}), \tag{2}$$

where the rate vector $\boldsymbol{x} = (x_1^1, \ldots, x_1^{K_1}, \ldots, x_N^1, \ldots, x_N^{K_N})$ denotes the transmission rates of all agents on all possible routes. Thus, the goal of the network manager is to solve the following optimization problem, called joint flow control and multipath routing problem (**FR**):

$$\begin{aligned} \max_{\boldsymbol{x}} \quad & \sum_{i \in \mathcal{N}} V_i(x_i^1, \ldots, x_i^{K_i}) \\ \text{s.t.} \quad & \sum_{i \in S_l} \sum_{k: l \in R_i^k} x_i^k \le c_l \quad \forall l \in L \\ & x_i^k \ge 0 \quad \forall i \in \mathcal{N} \quad \forall k = 1, \ldots, K_i, \end{aligned} \tag{\textbf{FR}}$$

where S_l is the set of agents having link l in at least one of their routes.

From Assumption 1, it follows that the objective function of **FR** is strictly concave. Moreover, the set of feasible transmission rates is nonempty, convex, and compact. Therefore, **FR** has a unique solution that we denote here by $\boldsymbol{x}^*$, and refer to it as the *social-welfare maximizing/efficient outcome*.

The network manager's problem would be a standard convex optimization problem, which can be solved by any standard convex optimization method, if the manager knew the valuation functions V_i, for each $i \in \mathcal{N}$. However, the agents' valuation functions are their own private information. Thus, in order to assign/allocate efficient transmission rates along each route for each agent, the network manager has to elicit information about each agent's valuation function. Since all agents are selfish (strategic) and want to maximize their own utility given by (1), they do not voluntarily reveal their information to the manager. Therefore, the manager needs to design a mechanism so as to align each agent's objective with his own objective. In the following section, we formulate the network manager's problem as a mechanism design problem. Then, in Section IV, we present an incentive mechanism to solve the problem.

III. Mechanism Design Problem

An incentive mechanism specifies the set of messages that each agent can use to transmit information, along with the decisions made based on the transmitted messages. Once a mechanism is in place, it induces a game among strategic agents who are competing for the network's resources. The network manager's problem is to design a mechanism so as to optimize the social welfare at all equilibria of the induced game.

Specifically, a mechanism is described by a tuple $\Gamma = (M_1, \ldots, M_N, g)$. In this tuple, M_i, $i \in \mathcal{N}$, specifies the set of messages agent i is allowed to broadcast, and $g(.)$ is the outcome function that determines the outcome $g(\boldsymbol{m})$ for any given message profile $\boldsymbol{m} := (m_1, \ldots, m_N) \in M_1 \times \ldots \times M_N$, where $m_i \in M_i$ denotes the message announced by agent i. We denote the mechanism's message space by $M := M_1 \times \ldots \times M_N$. For any $\boldsymbol{m} \in M$, the outcome function $g(.)$ takes the form $g(\boldsymbol{m}) = (y_i(\boldsymbol{m}), t_i(\boldsymbol{m}), i \in \mathcal{N})$, where the allocation function of agent i, $y_i : M \to \mathbb{R}_+^{K_i}$, determines the transmission rates of agent i through all her available paths, and the payment function of agent i, $t_i : M \to \mathbb{R}$, $i \in \mathcal{N}$, determines the monetary payment that agent i must make based on the message profile $\boldsymbol{m} \in M$. The mechanism is designed by the network manager. After the network manager designs and announces the mechanism he commits to it.

A mechanism $\Gamma = (M_1, \ldots, M_N, g)$, together with the utility functions U_i, $i \in \mathcal{N}$, induce a game $(M_1, \ldots, M_N, g, (U_i)_{i \in \mathcal{N}})$ among the agents, where $U_i(g(\boldsymbol{m}))$ is the utility of agent i resulting from the message profile $\boldsymbol{m}$. Because the description of each agent's private information is non-Bayesian, an appropriate solution concept for this game is the Nash equilibrium (NE). A NE is defined as a message profile $\boldsymbol{m}^*$ such that

$$U_i(g(m_i^*, \boldsymbol{m}_{-i}^*)) \geq U_i(g(m_i, \boldsymbol{m}_{-i}^*)) \quad \forall m_i \in M_i \quad \forall i \in \mathcal{N}, \tag{3}$$

where $\boldsymbol{m}_{-i}^* := (m_1^*, \ldots, m_{i-1}^*, m_{i+1}^*, \ldots, m_N^*)$ is the message profile of all agents except i. In words, the message profile $\boldsymbol{m}^*$ is a NE if for each $i \in \mathcal{N}$, the message m_i^* maximizes the utility of agent i, given the messages $\boldsymbol{m}_{-i}^*$ of all other agents.

The network manager's objective is to design a mechanism so as to *strongly implement* the social-welfare maximizing outcome defined by the solution of problem **FR** in NE. Strong implementation means that the allocations corresponding to *all* NE of the game induced by the mechanism are equal to the optimal solution of problem **FR**. The notion of strong implementation, in contrast to *truthful* or *partial implementation*, asserts that *every* equilibrium behavior by the agents delivers an outcome consistent with the network manager's objective. Therefore, if the game induced by the mechanism has at least one NE, the network manager no longer needs to worry about the strategic interactions of the agents.

For successful implementation, the network manager also needs to ensure that the equilibrium outcomes can be implemented without monetary transfers to or from the network. This requirement called budget balance (BB), can be written as

$$\sum_{i \in \mathcal{N}} t_i(\boldsymbol{m}^*) = 0 \quad \forall \text{ NE } \boldsymbol{m}^*. \tag{4}$$

Another desirable feature of a mechanism is the agents' voluntarily participation in the mechanism. Agent i voluntarily participates in the mechanism Γ if the utility she gets at any NE is at least as much as the "reservation utility" she gets when she does not participate in the mechanism. If agent i opts out of the mechanism she makes no payment and is not allowed to transmit any data through the network; thus, her reservation utility is $U_0^i = V_i(\mathbf{0}) = 0$, where the last equality follows from Assumption 1. Then agents' voluntarily participation is ensured by the following IR constraints as follows:

$$U_i(g(\boldsymbol{m}^*)) \geq 0 \quad \forall i \in \mathcal{N} \quad \forall \text{ NE } \boldsymbol{m}^*. \tag{5}$$

Furthermore, for the mechanism's practical implementation, the network manager must specify how strategic agents with asymmetric information can find a NE of the game induced by the mechanism in a decentralized manner. While there can be different formalizations of this question, in most cases a truly satisfactory answer would be to assume that the induced game is played repeatedly so as to allow each agent to learn about her rivals' strategies and find her own equilibrium strategy. The learning process is characterized by a tatonnement/iterative process (see [39]) that describes how agents iteratively update their messages using their prior information about the game and the information received at previous iterations. We call a mechanism Γ *dynamically stable* if there is a tatonnement process $\langle\mu\rangle$ that always converges to a NE of the game induced by the mechanism when the strategic agents update their messages according to $\langle\mu\rangle$. A dynamically stable mechanism ensures that there is a practical way to reach the outcome of an equilibrium of the game induced by it. Thus, dynamic stability is a desirable property of realistic mechanisms.

In summary, the problem the network manager faces is to design an incentive mechanism that possesses the following properties.

(P1) ***Existence of NE*:** The game induced by the mechanism has at least one NE.

(P2) ***Strong implementation*:** The allocations corresponding to every NE of the game induced by the mechanism are equal to the solution of problem **FR**.

(P3) ***Budget balance*:** The equilibrium outcome is implementable without monetary transfers to or from the network.

(P4) ***Individual rationality*:** Agents voluntarily participate in the mechanism.

(P5) ***Dynamic stability*:** The equilibrium outcome is reachable by a tatonnement process $\langle\mu\rangle$ when all agents follow $\langle\mu\rangle$.

The design of resource allocation mechanisms that possess properties (P1)–(P5) and are appropriate for networks with multiple links is a formidable problem. The mechanism presented in [30] possesses properties (P1)–(P5) but is only suitable for the allocation of a single good (single capacity). Our problem deals with the allocation of many goods (the capacities of many links). Furthermore, the allocations of these goods are not independent, because the rate an agent receives along every link through each of her routes must be the same. Therefore, allocations along the links of an agent's routes must be done in a coordinated manner, and this results in a resource allocation problem with multiple coupled goods that cannot be solved by running separate mechanisms for each link.

In this paper, we present an incentive mechanism that possesses properties (P1)–(P5) and solves the resource allocation problem formulated in this section. The mechanism utilizes the finite dimensional messages sent by the agents to approximate the private valuation function of each agent by a *surrogate valuation function*, and then maximizes the sum of surrogate functions under the network constraints. By adopting the right message space and payment policy, the mechanism incentivizes agents to send messages that make the derivative of their surrogate functions with respect to transmission rates equal to their marginal utilities. As a result, maximizing the sum of surrogate functions is equivalent to maximizing the sum of actual valuation functions, which are the agents' private information.

Besides the main properties (P1)–(P5), the mechanism we propose has some additional features that facilitate its implementation.

(S1) *Uniqueness of the NE*: When the efficient allocation is an interior point, the game induced by the mechanism has a unique NE. Uniqueness of NE is a desirable feature as it eliminates the issue of equilibrium selection.

(S2) *Feasibility out of equilibrium*: At all outcomes of the game induced by the mechanism, both at and off equilibrium, the allocation assigned to the agents is feasible, i.e., satisfies all the network constraints. Feasibility off-equilibrium is an essential requirement when an iterative process is employed for convergence to a NE point, as it ensures that allocations are still feasible when the iterative process has not yet converged.

IV. Specification of the Mechanism

We present a *dynamically stable surrogate optimization* (DSSO) mechanism that strongly implements the social-welfare maximizing outcome in NE, guarantees NE existence, satisfies the BB and IR constraints (4) and (5), and provides an algorithm for reaching an equilibrium. We first describe informally the idea behind our mechanism in Section IV-A and then go on to the formal description in Section IV-B.

A. Idea Behind Our Mechanism

The network manager's objective is to maximize the social welfare function $W(\boldsymbol{x}) = \sum_{i\in\mathcal{N}} V_i(x_i^1,\ldots,x_i^{K_i})$ defined in (2), under the network's constraints (problem **FR**), when $V_i(.)$, $i\in\mathcal{N}$, is agent i's private information. Since for practical reasons the network manager wishes to design a mechanism with finite-dimensional message space, he cannot ask the strategic agents to precisely report their valuation functions $V_i(.)$, as the space of all possible valuation functions for agent i, i.e., $\mathcal{V}_i$, $i\in\mathcal{N}$, is an infinite-dimensional space. As an alternative, the network manager may approximate the valuation function of each agent based on a finite number of parameters that the agent can report, and then maximize the sum of the approximated valuation functions.

With this idea in mind, the network manager chooses a real-valued function $f:\mathcal{R}_+\to\mathcal{R}$, announces it to the agents, and asks each agent i, $i\in\mathcal{N}$, to report the following:

1) K_i strictly positive weights $w_i^1,\ldots,w_i^{K_i}$, such that $\sum_{k=1}^{K_i} w_i^k f(x_i^k)$ best approximates her valuation function $V_i(x_i^1,\ldots,x_i^{K_i})$ in a sense that will be explained in Section V;
2) the maximum demand/rate the agent wants along each of her available routes; and
3) the price per unit of bandwidth/rate the agent is willing to pay at every link she is competing for.

Once the agents provide the above-mentioned reports, the network manager considers $\sum_{k=1}^{K_i} w_i^k f(x_i^k)$ as the *surrogate valuation function* of agent i and maximizes the sum of surrogate valuation functions under the constraints of problem **FR** taking into account the maximum demands agents want along each of their available routes. This maximization problem is called **OUT**.

To incentivize agents to send social-welfare supporting messages, the network manager must design monetary incentives that align each agent's objective with the social objective. One way to do this is to utilize the Lagrange multipliers associated with the optimal solution of problem **OUT** to determine the per-unit price of bandwidth per each link, so that the total payment of each agent over all links internalizes her effect on the social welfare. However, since Lagrange multipliers of problem **OUT** depend on all agents' messages, the network manager cannot directly set the price of each link equal to its corresponding Lagrange multiplier, because this creates incentive for strategic agents to manipulate their messages to pay less. To fix this, the network manager proceeds in the following two stages: (1) He asks agents to suggest a per-unit price of bandwidth for each link they are competing for, then uses the prices proposed by all users competing for a link (say l) except agent i to form a price per unit of bandwidth of link l for agent i; this price is independent of agent i's message. 2) He incentivizes agents to propose the Lagrange multipliers of problem **OUT** as their suggested unit prices. By doing so, the network manager indirectly makes the unit price of each link equal to its corresponding Lagrange multiplier. The monetary payments derived by taking this approach internalize the effect of each agent on the social welfare, hence they incentivize agents to send social-welfare supporting messages at every equilibrium.

The above-mentioned feature of the mechanism can guarantee strong implementation (P2); however, to induce a game that has a unique NE, the network manager charges each agent i an extra tax ψ_i that encourages agent i to act greedily in proposing maximum demands. With these ideas in mind, we formally specify our mechanism in the following section.

B. Formal Description of the DSSO Mechanism

The description of our mechanism $\Gamma=(M_1,\ldots,M_N,g)$ is divided into two parts: The message space and the outcome function.

1) The Message Space: The message space is

$$M = M_1 \times M_2 \times \ldots \times M_N, \tag{6}$$

where M_i defines the set of possible messages of agent i, and each $m_i\in M_i$ has the following form:

$$m_i = (\boldsymbol{w}_i, \boldsymbol{z}_i, \boldsymbol{p}_i). \tag{7}$$

Each message of agent i, m_i, is composed of three vectors: The weight vector

$$\boldsymbol{w}_i := (w_i^1,\ldots,w_i^{K_i}), w_i^k > 0, k=1,\ldots,K_i, \tag{8}$$

the maximum demand vector

$$\boldsymbol{z}_i := (z_i^1,\ldots,z_i^{K_i}), z_i^k \in [0, c_i^k], k=1,\ldots,K_i, \tag{9}$$

and the price per-unit of bandwidth vector

$$\boldsymbol{p}_i := (p_i^l : l\in\hat{R}_i), p_i^l \geq 0, l\in\hat{R}_i, \tag{10}$$

where z_i^k denotes agent i's maximum bandwidth demanded along her kth route, $c_i^k=\min_{l\in R_i^k} c_l$ denotes the maximum feasible rate along the kth route of agent i, $\hat{R}_i$ denotes the set of links that are part of the routes of agent i and are used by at least two agents, and p_i^l denotes the price agent i is willing to pay per unit of bandwidth at link l, when l is used by two or more agents. We call "competitive links" all links used by two or more agents.

2) Outcome Function: The outcome function g of the DSSO mechanism

$$g : M \rightarrow \mathbb{R}_+^{\sum_{i=1}^N K_i} \times \mathbb{R}^N, \tag{11}$$

determines the outcome $g(\boldsymbol{m})$ for any given message profile $\boldsymbol{m} \in M$. For each $\boldsymbol{m} \in M$, the outcome function $g(.)$ takes the form $g(\boldsymbol{m}) = (y_i(\boldsymbol{m}), t_i(\boldsymbol{m}), i \in \mathcal{N})$, where the allocation function for agent i, $y_i : M \rightarrow \mathbb{R}_+^{K_i}$, determines the bandwidth allocated to i along all her available paths, and the payment function of agent i, $t_i : M \rightarrow \mathbb{R}$, $i \in \mathcal{N}$, determines the monetary payment i makes based on the message profile $\boldsymbol{m} \in M$. The allocation and payment functions are determined as follows.

Allocation function: At the beginning of the mechanism, the network manager selects a function $f : \mathcal{R}_+ \rightarrow \mathcal{R}$, which he announces to the strategic agents; $f(.)$ could be any continuous, differentiable, strictly concave, and increasing function with $f'(0) < \infty$. Then, based on the agents' messages, the network manager forms a surrogate valuation function $\sum_{k=1}^{K_i} w_i^k f(x_i^k)$ for each agent $i \in \mathcal{N}$, and maximizes the sum of surrogate functions under the network and demand constraints according to problem **OUT** stated below. Specifically, we define the allocation function $y(.) = (y_i(.), i \in \mathcal{N})$, as the function that maps each message profile $\boldsymbol{m}$ to the solution of the following optimization problem, called **OUT**:

$$\begin{aligned} \max_{\boldsymbol{x}} \quad & \sum_{i \in \mathcal{N}} \sum_{k=1}^{K_i} w_i^k f(x_i^k) \\ \text{s.t.} \quad & \sum_{i \in S_l} \sum_{k: l \in R_i^k} x_i^k \leq c_l \quad \forall l \in L \\ & x_i^k \geq 0 \quad \forall i \in \mathcal{N} \quad \forall k = 1, \ldots, K_i \\ & x_i^k \leq z_i^k \quad \forall i \in \mathcal{N} \quad \forall k = 1, \ldots, K_i. \end{aligned} \tag{OUT}$$

This problem always has a unique solution, therefore, the allocation function $y(.)$ is well defined.

For each message profile $\boldsymbol{m} = (\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{p})$, the problem **OUT** and its unique solution depend only on the weights and maximum demands announced by the agents, and not on the announced prices. Therefore, for a given set of weights $\boldsymbol{w} = (w_i^k : i \in \mathcal{N}, k = 1, \ldots, K_i)$ and a given set of maximum demands $\boldsymbol{z} = (z_i^k : i \in \mathcal{N}, k = 1, \ldots, K_i)$, we denote the unique solution of problem **OUT** by $y(\boldsymbol{w}, \boldsymbol{z}) = (y_i(\boldsymbol{w}, \boldsymbol{z}) : i \in \mathcal{N})$, where $y_i(\boldsymbol{w}, \boldsymbol{z})$ specifies the transmission rate/bandwidth allocated to agent i in each of her available paths, when agents sent $\boldsymbol{w}$ and $\boldsymbol{z}$ as part of their messages.

Payment function: Given the message profile $\boldsymbol{m} = (\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{p})$, we define the tax $t_i(\boldsymbol{m})$ to be paid by each agent i as follows:

$$t_i(\boldsymbol{m}) = \sum_{l \in \hat{R}_i} t_i^l(\boldsymbol{m}) + \psi_i(\boldsymbol{w}, \boldsymbol{z}). \tag{12}$$

The tax $t_i^l(\boldsymbol{m})$ that must be paid by agent i for usage of competitive link l is defined by

$$t_i^l(\boldsymbol{m}) = P_{-i}^l \left(\sum_{k: l \in R_i^k} y_i^k(\boldsymbol{w}, \boldsymbol{z}) - \frac{1}{|S_l|} c_l \right) + (p_i^l - \lambda_l(\boldsymbol{w}, \boldsymbol{z}))^2, \tag{13}$$

where P_{-i}^l is the average of the prices proposed for link l by all agents competing to use l other than agent i, i.e.,

$$P_{-i}^l = \frac{1}{|S_l| - 1} \sum_{j \in S_l, j \neq i} p_j^l, \tag{14}$$

and $\boldsymbol{\lambda}(\boldsymbol{w}, \boldsymbol{z}) = (\lambda_l(\boldsymbol{w}, \boldsymbol{z}) : l \in L)$ denotes the centroid of the set of Lagrange multiplier vectors of problem **OUT** corresponding to the capacity constraints, when the set of weights $\boldsymbol{w}$ and the set of maximum demands $\boldsymbol{z}$ are announced by the agents. The centroid of a set of vectors, which is also called the center of mass, is the vector whose coordinates are the mean values of the coordinates of all the vectors in the set.

The first term of the tax $t_i^l(\boldsymbol{m})$ is the tax paid (or received) by agent i for her transmission rate $\sum_{k: l \in R_i^k} y_i^k(\boldsymbol{w}, \boldsymbol{z})$ over link l, that is in excess (or short) of the fair share of agents over this link. This tax is based on a price per unit of bandwidth determined by the prices proposed by the agents competing for link l except agent i. Therefore, agent i has no control over the price P_{-i}^l, and hence, has no incentive to manipulate her message to reduce it. The second term of the tax $t_i^l(\boldsymbol{m})$ is a penalty imposed on agent i for announcing a price p_i^l that is different from $\lambda_l(\boldsymbol{w}, \boldsymbol{z})$. As the result of this monetary incentive, the per unit price of each link at equilibrium will be uniform and equal to its corresponding Lagrange multiplier in problem **OUT** for all agents.

The second term on the right-hand side of (12) is given by

$$\psi_i(\boldsymbol{w}, \boldsymbol{z}) = \begin{cases} 1, & \text{if } \exists k : z_i^k \neq c_i^k, \exists \boldsymbol{w}_i' > 0 : y_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}), \\ & \quad (c_i^1, \ldots, c_i^{K_i}, \boldsymbol{z}_{-i})) = y_i(\boldsymbol{w}, \boldsymbol{z}) \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

The tax $\psi_i(\boldsymbol{w}, \boldsymbol{z})$ is nonzero when the two following conditions are met simultaneously: 1) there exists at least one route $R_i^k, k = 1, \ldots, K_i$, along which the maximum bandwidth z_i^k demanded by agent i is different from the maximum feasible rate c_i^k; and 2) there exists another weight vector $\boldsymbol{w}_i' > 0$ such that agent i can achieve the same rate as $y_i(\boldsymbol{w}, \boldsymbol{z})$, by changing her announced weight vector from $\boldsymbol{w}_i$ to $\boldsymbol{w}_i'$ and demanding the maximum feasible rates c_i^k, $k = 1, \ldots, K_i$, along all her paths. Therefore, this monetary incentive $\psi_i(\boldsymbol{w}, \boldsymbol{z})$ encourages agent i to act in a greedy way and ask for maximum feasible rates along all her paths, i.e., $z_i^k = c_i^k$, for all $k = 1, \ldots, K_i$, whenever it does not prevent her from achieving her desired rates. As we discussed in Section IV-A, the role of this incentive is to induce games with a unique NE.

V. Properties of the DSSO Mechanism

We prove that the DSSO mechanism proposed in Section IV has main properties (P1)–(P4) and additional features (S1)–(S2) stated in Section III. We postpone proving property (P5) to the next section.

We establish these properties by proceeding as follows. We show that although the DSSO mechanism is a discriminatory price mechanism off-equilibrium, i.e., it sells the bandwidth of each link l at different per-unit prices P^l_{-i} to different agents i, it is a uniform price mechanism at equilibrium, i.e., the equilibrium per-unit price of each link l is uniform and equal to the Lagrange multiplier $\lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$ for all agents (see Lemma 1). We use this property to show that the mechanism is budget-balanced at all Nash equilibria of the game induced by it (see Theorem 1).

We prove that at each message profile, each agent is able to unilaterally deviate to any feasible transmission rates over her available paths (see Lemma 3). We use this property to show that the DSSO mechanism strongly implements the solution of the flow control and routing problem **FR** in Nash equilibria (see Theorem 2). We show that at all Nash equilibria of the game induced by the mechanism the agents' utilities are nonnegative, i.e., they are weakly preferred to 0, the reservation utility agents get when they do not participate in the mechanism. This result establishes IR (see Theorem 3). Afterward, we prove the existence of NE at each game induced by the mechanism (see Theorem 4) and show that in most cases the NE message is unique (see Theorem 5). Finally, we prove the feasibility of allocations both at and off equilibrium (see Theorem 6).

We present the proofs of the following theorems and lemmas in the Appendix.

Lemma 1: Let $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ be a NE of the game induced by the DSSO mechanism. Then, for every agent $i \in \mathcal{N}$, and every competitive link l in the routes of agent i, we have

$$p_i^{l*} = P_{-i}^{l*} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*). \quad (16)$$

As a result of Lemma 1, the penalty tax $(p_i^l - \lambda_l(\boldsymbol{w}, \boldsymbol{z}))^2$ [second term of (13)] is equal to zero for each agent $i \in \mathcal{N}$ at every NE. The following Lemma shows that the same result holds for the other penalty tax ψ_i (15) that must be paid by agent i.

Lemma 2: Let $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ be a NE of the game induced by the DSSO mechanism. Then, for every agent $i \in \mathcal{N}$, we have

$$\psi_i(\boldsymbol{w}^*, \boldsymbol{z}^*) = 0. \quad (17)$$

An immediate consequence of Lemmas 1 and 2 is the following. At every NE $\boldsymbol{m}^*$ of the game induced by the mechanism, the payment function has the following form:

$$t_i(\boldsymbol{m}^*) = \sum_{l \in \hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \left(\sum_{k: l \in R_i^k} y_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*) - \frac{1}{|S_l|} c_l \right). \quad (18)$$

We use this result to show that the total payments of all agents at equilibrium is zero. This proves that at equilibrium the proposed mechanism is budget balanced.

Theorem 1 (BB): Consider any NE $\boldsymbol{m}^*$ of the game induced by the DSSO mechanism. Then,

$$\sum_{i \in \mathcal{N}} t_i(\boldsymbol{m}^*) = 0. \quad (19)$$

The next lemma states an important property of the DSSO mechanism that is key to proving strong implementation and IR.

Lemma 3: Consider agent $i \in \mathcal{N}$; given the messages of all other agents except i, i.e., $\boldsymbol{m}_{-i}$, each feasible vector of transmission rates $\boldsymbol{x}_i = (x_i^1, \ldots, x_i^{K_i})$ for agent i is achievable. More specifically, given any vector $\boldsymbol{x}_i = (x_i^1, \ldots, x_i^{K_i})$ such that $\sum_{k: l \in R_i^k} x_i^k \leq c_l$, and $x_i^k \geq 0$, for all $k = 1, \ldots, K_i$, there exists a message $\boldsymbol{m}_i = (\boldsymbol{w}_i, \boldsymbol{z}_i, \boldsymbol{p}_i)$ such that $y_i(\boldsymbol{w}, \boldsymbol{z}) = \boldsymbol{x}_i$, $\psi_i(\boldsymbol{w}, \boldsymbol{z}) = 0$, and $p_i^l = \lambda_l(\boldsymbol{w}, \boldsymbol{z})$ at all links agent i competing for bandwidth.

As a result of Lemma 3, agent i can unilaterally deviate from a NE point $\boldsymbol{m}^*$ to achieve any feasible transmission rate vector $(x_i^1, \ldots, x_i^{K_i})$, without paying any penalty tax. In these deviations, the per-unit price of each link l agent i competing for remains constant and equal to the equilibrium price, i.e., $P_{-i}^l = P_{-i}^{l*} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$, because these prices are determined based on the other agents' messages that are unchanged. Therefore, agent i can achieve any utility of the form

$$V_i(x_i^1, \ldots, x_i^{K_i}) - \sum_{l \in \hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \left(\sum_{k: l \in R_i^k} x_i^k - \frac{1}{|S_l|} c_l \right), \quad (20)$$

where $\sum_{k: l \in R_i^k} x_i^k \leq c_l$ and $x_i^k \geq 0$, for all $k = 1, \ldots, K_i$, by deviating from $\boldsymbol{m}^*$. Since $\boldsymbol{m}^*$ is a NE point, agent i must have no incentive to unilaterally deviate; therefore, the maximum of the above-mentioned expression (20) must be attained at $(x_i^1, \ldots, x_i^{K_i}) = y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)$, i.e.,

$$y_i(\boldsymbol{w}^*, \boldsymbol{z}^*) =$$

$$\arg\max_{\boldsymbol{x_i}} V_i(x_i^1, \ldots, x_i^{K_i}) - \sum_{l \in \hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \left(\sum_{k: l \in R_i^k} x_i^k - \frac{1}{|S_l|} c_l \right)$$

$$\text{s.t.} \quad \sum_{k: l \in R_i^k} x_i^k \leq c_l \quad \forall l \in L$$

$$x_i^k \geq 0 \quad \forall k = 1, \ldots, K_i. \qquad \mathbf{NASH}_i$$

As a result, at every NE point $\boldsymbol{m}^*$, the transmission rates of each agent i, $y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)$ along with a Lagrange multiplier vector satisfies the KKT conditions of the optimization problem mentioned above called $\mathbf{NASH}_i$. We use this result to show that at every NE point $\boldsymbol{m}^*$, there exists a Lagrange multiplier vector for problem **FR** along which the transmission rate vector $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*)$ satisfies the KKT conditions of problem **FR** and hence is optimal.

Theorem 2 (Strong Implementation): At any NE $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ of the game induced by the DSSO mechanism, the rate vector $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*)$ is equal to the unique optimal solution of problem **FR**, $\boldsymbol{x}^*$.

Theorem 2 asserts that, unlike other mechanisms that use surrogate optimization [9], [10], the DSSO mechanism satisfies the strong implementation property. A key feature of the DSSO mechanism that leads to strong implementation of social-welfare maximizing outcome defined by problem **FR** is the property established in Lemma 3. In the absence of this prop-

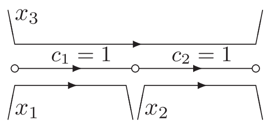

Fig. 1. Network scenario considered in Example 1.

erty, the rate $\boldsymbol{y}_i(\boldsymbol{w}^*, \boldsymbol{z}^*)$ assigned to an agent i at a NE point could be different from the optimal rate that maximizes the valuation function of agent i over all feasible transmission rates, in which case Theorem 2 would not hold. The validity of Lemma 3 in turn hinges upon the inclusion of the maximum demand rates $\boldsymbol{z}_i$ in the agent messages and the corresponding constraints in problem **OUT**, a feature lacking in previous surrogate mechanisms [9], [10]. The important role played by the maximum demand rates $\boldsymbol{z}_i$ in establishing Lemma 3 and Theorem 2 is illustrated by the following example.

Example 1: Consider a mechanism similar to the DSSO, except that agents do not announce a maximum demand rate as part of their messages. In this mechanism, the message of each agent i is of the form $m_i = (\boldsymbol{w}_i, \boldsymbol{p}_i)$, the constraint involving $\boldsymbol{z}_i$ is omitted from **OUT**, and the payment term $\psi_i(\boldsymbol{w}, \boldsymbol{z})$ is omitted from (12). We show that in this mechanism neither Lemma 3 nor Theorem 2 remain valid. To this end consider the network shown in Fig. 1 , where each agent $i = 1, 2, 3$ has only one available route, and the valuation functions of agents are as follows:

$$V_1(x_1) = 0.3 \ln(x_1 + 1), V_2(x_2) = 4 \ln(x_2 + 1),$$
$$V_3(x_3) = 2.5 \ln(x_3 + 1). \tag{21}$$

With these valuation functions, the optimal solution of problem **FR** is $\boldsymbol{x}^* = (0.897, 0.897, 0.103)$. Now, consider $f(x) = \ln(x + 1)$. By some algebra, we can show that the message profile $\boldsymbol{m} = (m_1, m_2, m_3)$, where

$$m_1 = (w_1, p_1^1) = (1, 0.5), m_2 = (w_2, p_2^2) = (4, 2),$$
$$m_3 = (w_3, p_3^1, p_3^2) = (2, 0.5, 2), \tag{22}$$

is a NE for the game induced by the mechanism described above. This NE results in the rate vector $\boldsymbol{y} = (1, 1, 0)$, which is not equal to $\boldsymbol{x}^*$; thus this mechanism does not strongly implement $\boldsymbol{x}^*$ in NE. The reason is the following. Consider agent 1; by unilaterally deviating from m, this agent cannot achieve any rate other than $x_1 = 1$ (leading to violation of Lemma 3). Had agent 1 been allowed to announce a maximum demand rate z_1, any rate $x_1 \in [0, 1]$ could have been achieved simply by letting $z_1 = x_1$. In this way, outcomes like $\boldsymbol{y} = (1, 1, 0)$ could have been avoided by agent 1.

Another consequence of Lemma 3 is IR. As we discussed before, Lemma 3 implies that the utility of each agent i at every NE $\boldsymbol{m}^*$ must be greater than or equal to any utility of the form (20), where $(x_i^1, \ldots, x_i^{K_i})$ is a feasible transmission rate vector for agent i. One of such feasible transmission rate vectors is a zero vector $(x_i^1, \ldots, x_i^{K_i}) = (0, \ldots, 0)$ in which agent i does not transmit any data through the network. Therefore, we must have

$$U_i(g(\boldsymbol{m}^*)) \geq V_i(0, \ldots, 0) - \sum_{l \in \hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$$
$$\left(\sum_{k: l \in R_i^k} 0 - \frac{1}{|S_l|} c_l \right) = \sum_{l \in \hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \left(\sum_{k: l \in R_i^k} \frac{1}{|S_l|} c_l \right) \geq 0, \tag{23}$$

for every NE $\boldsymbol{m}^*$, which proves that the DSSO mechanism satisfies the IR constraint (5).

Theorem 3 (IR): The DSSO mechanism is individually rational, that is, each agent weakly prefers the outcome of every NE of the game induced by the DSSO mechanism to the reservation utility she gets when she does not participate in the resource allocation and routing process.

The following result asserts the existence of NE in the game induced by the DSSO mechanism.

Theorem 4 (Existence of NE): Let $\boldsymbol{x}^* = (\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_N^*)$ be the unique optimal solution of **FR**. Then, the message profile $\boldsymbol{m}^* = (m_i^*, i \in \mathcal{N})$ with each message m_i^* defined by

$$m_i^* = ((w_i^{*k})_{\forall k}, (z_i^{*k})_{\forall k}, (p_i^{*l})_{\forall l \in \hat{R}_i}), \tag{24}$$

where

$$w_i^{*k} = \frac{\left.\frac{\partial V_i}{\partial x_i^k}\right|_{\boldsymbol{x}_i^*}}{f'(x_i^{k*})}, z_i^{*k} = c_i^k, p_i^{*l} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*), \tag{25}$$

is a NE of the game induced by the DSSO mechanism.

At the NE message defined by (24) and (25), each agent i asks for maximum demand along all her routes $z_i^{*k} = c_i^k$, and announces weights that make the derivative of her surrogate function with respect to transmission rates equal to her marginal utilities, i.e., $w_i^{*k} f'(x_i^{k*}) = \left.\frac{\partial V_i}{\partial x_i^k}\right|_{\boldsymbol{x}_i^*}$. Therefore, maximization of the sum of surrogate functions is equivalent to the maximization of the sum of actual valuation functions and results in a social-welfare maximizing outcome. The following result shows that in most of the cases, the NE message defined by (24) and (25) is the only NE of the game induced by the mechanism.

Theorem 5 (Uniqueness): If $0 < x_i^{k*} < c_i^k$, for all $i \in \mathcal{N}$, and $k = 1, \ldots, K_i$, then the message profile $\boldsymbol{m}^* = (m_i^* : i \in \mathcal{N})$ defined in Theorem 4, is the unique NE of the game induced by the DSSO mechanism.

Theorem 5 proves that the DSSO mechanism satisfies feature (S1). The next theorem proves the satisfaction of property (S2), by showing that the outcome of the game induced by the DSSO mechanism is always feasible, even off equilibrium.

Theorem 6 (Feasibility off equilibrium): For any set of messages $\boldsymbol{m} = (\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{p})$, the rate vector $\boldsymbol{y}(\boldsymbol{w}, \boldsymbol{z})$ is a feasible point of problem **FR**.

In the next section, we prove the dynamic stability of the DSSO mechanism.

VI. Dynamic Stability

A mechanism Γ is dynamically stable if there exists a tatonnement/iterative process $\langle \mu \rangle$ with the following property. If in the game induced by Γ all agents update their messages according to $\langle\mu\rangle$, the sequence of outcomes generated by $\langle\mu\rangle$ converges to a NE. In this case, we say that the mechanism Γ is stable under the tatonnement process $\langle\mu\rangle$.

Formally, a tatonnement process is determined by an update rule $\mu : M \to M$, specifying a new message profile $\mu(\boldsymbol{m})$ for each previously announced message profile $\boldsymbol{m} \in M$. Let $\boldsymbol{m}^t$ denote the message profile announced by the agents in iteration $t, t \in \{0, 1, \ldots\}$. Then, if agents follow the tatonnement process $\langle\mu\rangle$, they will send message profile $\boldsymbol{m}^{t+1} = \mu(\boldsymbol{m}^t)$ in the next iteration $t + 1$.

In this section, we prove under certain conditions, the dynamic stability of the DSSO mechanism by constructing a tatonnement process under which the DSSO mechanism is stable. For that matter, we need the following technical assumption.

Assumption 2: For each link l there is at least one agent i such that $R_i^k = \{l\}$, for a $k \in \{1, \ldots K_i\}$.

The purpose of Assumption 2 is to ensure that all the links of the network are always saturated if the agents do not have limiting maximum demand constraints. This assumption is similar in spirit to the special buyer assumption made in [9]; however, it is weaker than that, because the special buyer assumption requires that at least two single-link agents use each link, and that the valuation functions of those single-link agents satisfy $U'(0) = \infty$. To obtain the stability results, we also restrict attention to a set of valuation functions that satisfy the following condition.

Assumption 3: All agents $i \in \mathcal{N}$ have additive separable valuation functions $V_i(x_i^1, \ldots, x_i^{K_i}) = \sum_{k=1}^{K_i} V_i^k(x_i^k)$, where the first and second derivatives of V_i^ks at point 0 satisfy

$$\left|V_i^{k'}(0) + bV_i^{k''}(0)\right| < \left(\frac{b}{c+b}\right)^{3(P-L)-1} \frac{q^{P-L}}{Q^{P-L-1}} S, \quad (26)$$

where $b = \min_{x\in[0,c_i^k]} \min\left(\frac{V_i^{k'}(x)}{|V_i^{k''}(x)|}, \frac{2|V_i^{k''}(x)|}{|V_i^{k'''}(x)|}\right) - x$ is a positive number, $P = \sum_{j\in\mathcal{N}} K_j$ is the total number of routes, $q = \min_{i,k} V_i^{k'}(0)$ is the minimum of the marginal utilities of agents at rate 0, and $Q = \max_{i,k} V_i^{k'}(c)$ is the maximum of the marginal utilities of agents at rate $c = \max_{l\in\mathcal{L}} c_l$. Moreover, S is a network index defined below.

Definition 1: The network index S is defined by

$$S = \min_{1\le a\le P} \frac{\left|\det(R^T R)\right|}{\sum_{b=1}^{P} \left|\det({R_{-a}}^T R_{-b})\right|}, \quad (27)$$

where R is the $P \times L$ binary routing matrix, with the $(\sum_{d=1}^{i-1} K_d + k, l)$ element equals to 1 if the lth link of the network is in the kth available route for agent i, and 0 otherwise. For each integer a, $1 \le a \le P$, R_{-a} denotes matrix R when row a is removed.

Assumption 3 puts a limit on the first- and second-order derivatives of the valuation functions of agents at point 0. Having bounds on marginal utility and concavity of the preferences can be justified by the earlier instability results by Jordan [40] and Kim [41]. These results imply that the stability cannot be achieved for completely general valuation functions. Moreover, Jordan remarks that the instability results crucially relies on the range of higher order preference/valuation function behavior present in the environment. Therefore, having a bound like (26) on the first- and second-order derivatives of the valuation functions at point 0 is not unexpected. The aim of this assumption is to eliminate or bound the undesirable higher order effects on stability. There are many classes of networks and valuation functions that satisfy Assumption 3 along with Assumption 2. The following example shows one of these classes.

Example 2: Consider a family of networks with L cascade links and $L + 1$ agents, where agent $1 \le i \le L$ has link i as her route, and agent $L + 1$ uses all the links to transmit data from her source to her destination. (See Fig. 1 for $L = 2$.) It can be shown that every network of this family has a network index of 1, i.e. $S = 1$. Assume that the agent's valuation functions are described by an increasing and strictly concave function of the form

$$V_i(x_i) = e_i \frac{x}{\gamma_i(x + \gamma_i)}, \quad (28)$$

where e_i and γ_i are arbitrary positive numbers. This family of networks and valuation functions satisfies Assumptions 2 and 3, if $\gamma_i \in (r, 2r)$, where r is much greater than the maximum capacity of links c.

Under Assumptions 2 and 3, using the concept of contraction mappings [42], we propose a tatonnnement process that establishes the dynamic stability of the DSSO mechanism. The idea of using contraction property to achieve dynamic stability was first exploited by Healy and Mathevet in [30]. They considered a single-link network and designed a mechanism that is contractive and hence stable under the BR tatonnement process. Subsequent attempts to design a mechanism for networks with many links, which is stable under the BR, were not successful. Therefore, in proving dynamic stability of the DSSO mechanism, instead of focusing on the BR, we design a new tatonnement process under which the DSSO mechanism is stable.

We propose our tatonnement process called Best-Estimate (BE) in Section VI-A, and then in Section VI-B, we prove that the DSSO mechanism is stable under BE.

A. BE Tatonnement Process

The BE tatonnement process $\langle\mu_{\mathrm{BE}}\rangle$ is an algorithm that agents use to search for a NE outcome. According to Theorems 4 and 5, the message profile $\boldsymbol{m}^* = (m_i^* : i \in \mathcal{N})$, with each message m_i^* defined in (24) and (25) is a typically unique NE of the game induced by the DSSO mechanism. Thus, each agent i knows the form of her equilibrium message $m_i^* = (\boldsymbol{w}_i^*, \boldsymbol{z}_i^*, \boldsymbol{p}_i^*)$, where $w_i^{*k} = \frac{\frac{\partial V_i}{\partial x_i^k}|_{\boldsymbol{x}_i^*}}{f'(x_i^{k*})}$, $z_i^{*k} = c_i^k$, and $p_i^{*l} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$; however, since any agent has no information about the valuation functions of the other agents, she cannot directly compute the social-welfare maximizing outcome $\boldsymbol{x}^*$ that is needed to determine her equilibrium weight vector. This lack of information also makes her unable to directly calculate her equilibrium prices $p_i^{*l} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$, because $\lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$ is a function of equilibrium weights of the other agents. A tatonnement process provides an iterative procedure through which agents compute NE messages. A rule of thumb strategy for agent i in iteration $t+1$ is to approximate $\boldsymbol{x}^*$ and $\lambda(\boldsymbol{w}^*, \boldsymbol{z}^*)$ by the solution $\boldsymbol{y}^{(t)} = \boldsymbol{y}(\boldsymbol{w}^{(t)}, \boldsymbol{z}^{(t)})$ and the Lagrange multiplier vector $\boldsymbol{\lambda}^{(t)} = \boldsymbol{\lambda}(\boldsymbol{w}^{(t)}, \boldsymbol{z}^{(t)})$ of problem **OUT** in the tth round. Therefore, the update rule of our proposed BE tatonnement process, denoted by $\langle \mu_{\mathrm{BE}} \rangle$, is

$$\boldsymbol{m}^{(t+1)} = \mu_{\mathrm{BE}}(\boldsymbol{m}^{(t)}) = (\boldsymbol{w}_i^{(t+1)}, \boldsymbol{z}_i^{(t+1)}, \boldsymbol{p}_i^{(t+1)})_{i \in \mathcal{N}}, \quad (29)$$

where

$$w_{i^k}^{(t+1)} = \frac{\partial V_i}{\partial x_i^k}\Big|_{\boldsymbol{y}_i^{(t)}} / f'(y_i^{k\,(t)}), \quad k = 1, \ldots, K_i$$
$$z_i^{k\,(t+1)} = c_i^k, \quad k = 1, \ldots, K_i$$
$$p_i^{l\,(t+1)} = \lambda_l^{(t)}, \quad l \in \hat{R}_i. \quad (30)$$

B. Analysis and Results

We prove that the DSSO mechanism is stable under the BE process $\langle \mu_{\mathrm{BE}} \rangle$ as follows. First, we show that in the game induced by the DSSO mechanism, the unique fixed point of the updating function μ_{BE} is a NE (see Lemma 4). Then, we prove that if Assumptions 2 and 3 are met, the network manager can always choose an appropriate surrogate function $f(.)$ so as to guarantee convergence of the BE process to the fixed point. To do so, we construct a compressed form of the update rule μ_{BE} called T defined by (31)–(32) below, such that in the game induced by the DSSO mechanism, the convergence of the BE process $\langle \mu_{\mathrm{BE}} \rangle$ is equivalent to the convergence of the Compressed BE (CBE) process $\langle T \rangle$. Then, we prove that if the network manager chooses a suitable surrogate function, the DSSO mechanism results in games in which the function T is a contraction mapping, and hence, the CBE process is globally convergent (see Lemma 5). In Theorem 6, we combine all above-mentioned results to show that the DSSO mechanism is dynamically stable.

Lemma 4: In the game induced by the DSSO mechanism, the unique fixed point of function μ_{BE} is the NE message profile defined by (24) and (25).

Lemma 4 shows that if the BE process converges, the point of convergence must be a NE. However, this lemma does not assert that the BE process necessarily converges. In the rest of this section, we show that the network manager can always guarantee convergence of the BE process by selecting a suitable surrogate function. To show this, we construct a compressed BE (CBE) tatonnement process $\langle T \rangle$, such that the convergence of the BE and CBE processes are equivalent. The construction of $\langle T \rangle$ is based on the two following observations: 1) In the update rule μ_{BE} defined by (29)–(30), $z_i^{k\,(t+1)}$, $k = 1, \ldots, K$, is a fixed quantity that only depends on the network structure and not on the announced messages. Therefore, from the second round on, the announced $\boldsymbol{z}_i$s, $i \in \mathcal{N}$, are always fixed and can be omitted from the convergence study. 2) Even though the price component $p_i^{l\,(t+1)}$ is a function of the previously announced message profile $\boldsymbol{m}^{(t)}$, it does not affect the announced messages of the subsequent rounds $\boldsymbol{m}^{(t')}$, $t' > t+1$. Therefore, it has no effect on the convergence study and can be neglected. Consequently for the convergence study, we can restrict attention to the tatonnement process $\langle T \rangle$ with the following update rule:

$$T(\boldsymbol{w}) = (T_i^k(\boldsymbol{w}))_{i \in \mathcal{N}, k=1,\ldots,K_i}, \quad (31)$$

where

$$T_i^k(\boldsymbol{w}) = \frac{\partial V_i}{\partial x_i^k}\Big|_{y_i(\boldsymbol{w},\boldsymbol{c})} / f'(y_i^k(\boldsymbol{w}, \boldsymbol{c})), \quad (32)$$

which only tracks the evolution of the weight vectors.

Lemma 5: Under Assumptions 2 and 3, the network manager can always choose a logarithmic surrogate function $f(x) = \log(x/\gamma + 1)$ with an appropriate parameter γ, so that after the second iteration of the CBE process, function T becomes a contraction mapping in the game induced by the DSSO mechanism.

An immediate consequence of Lemma 5 and the Banach theorem [42] is that by choosing an appropriate surrogate function $f(.)$ the network manager can make the CBE process globally convergent. Since the convergence of the BE and CBE processes are equivalent, we can use this equivalence along with the result of Lemma 4 to state the following theorem, which proves the dynamic stability of the DSSO mechanism (see property P5).

Theorem 7: Under Assumptions 2 and 3, the DSSO mechanism is stable under the BE process.

VII. Conclusion

Using a surrogate optimization approach, we proposed an incentive mechanism for joint flow control and multipath routing problem in networks with multiple links and strategic agents. We showed that the mechanism strongly implements the social-welfare maximizing solution in NE, it is budget balanced at equilibrium, it is individually rational, and it is dynamically stable. To establish the dynamic stability, we propose a BE tatonnement process and proved that it converges to a NE of the game induced by the mechanism provided that the strategic agents follow it. The design of a tatonnement process that converges to a NE of the game induced by the mechanism and ensures that all strategic agents voluntarily follow it is a challenging open problem that is left for our future research.

Appendix

A. Proof of Lemma 1

Consider a case in which all agents adhere to message profile $\boldsymbol{m}^*$, except for agent i who broadcasts message $m_i = (\boldsymbol{w}_i^*, \boldsymbol{z}_i^*, \boldsymbol{p}_i)$, with an arbitrary unit price vector $\boldsymbol{p}_i \in \mathbb{R}_+^{|\hat{R}_i|}$. Since $\boldsymbol{m}^*$ is a NE, by this deviation the utility of agent i should not increase, i.e.,

$$V_i(y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)) - t_i((\boldsymbol{w}_i^*, \boldsymbol{z}_i^*, \boldsymbol{p}_i), \boldsymbol{m}_{-i}^*)$$
$$\leq V_i(y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)) - t_i(\boldsymbol{m}_i^*, \boldsymbol{m}_{-i}^*) \quad \forall \boldsymbol{p}_i \in \mathbb{R}_+^{|\hat{R}_i|}. \quad (33)$$

Substituting (12) and (13) in (33), we get

$$\sum_{l \in \hat{R}_i} (p_i^{l*} - \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*))^2 \leq \sum_{l \in \hat{R}_i} (p_i^l - \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*))^2, \quad (34)$$

for all $\boldsymbol{p}_i \in \mathbb{R}_+^{|\hat{R}_i|}$. Thus, $\boldsymbol{p}_i^*$ is the optimal solution of the following problem:

$$\min_{\boldsymbol{p}_i \in \mathbb{R}_+^{|\hat{R}_i|}} \sum_{l \in \hat{R}_i} (p_i^l - \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*))^2. \tag{35}$$

Due to KKT conditions for problem **OUT**, we have $\lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \geq 0$ for all $l \in \hat{L}$. Thus, the optimal solution of (35) is

$$\boldsymbol{p}_i^* = (p_i^{l*} : l \in \hat{R}_i), p_i^{l*} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*). \tag{36}$$

Substituting (36) in (14) completes the proof. ■

B. Proof of Lemma 2

We prove this lemma by contradiction. Suppose there exists a NE $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ such that $\psi_i(\boldsymbol{w}^*, \boldsymbol{z}^*) = 1$, for $i \in \mathcal{N}$. Then, by definition (15), there exists a $z_i^{k*} \neq c_i^k$, and there exists a positive $\boldsymbol{w}_i'$ such that

$$y_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}^*), (\boldsymbol{c}_i, \boldsymbol{z}_{-i}^*)) = y_i(\boldsymbol{w}^*, \boldsymbol{z}^*), \tag{37}$$

where $\boldsymbol{c}_i = (c_i^1, \ldots, c_i^{K_i})$. Now, consider a deviation of agent i from m_i^* to $m_i = (w_i', \boldsymbol{c}_i, \boldsymbol{p}_i)$ with $p_i^l = \lambda_l((w_i', \boldsymbol{w}_{-i}^*), (\boldsymbol{c}_i, \boldsymbol{z}_{-i}^*))$. Since $\boldsymbol{m}^*$ is NE, we must have

$$U_i(g(m_i, \boldsymbol{m}_{-i}^*)) \leq U_i(g(\boldsymbol{m}^*)). \tag{38}$$

Substituting (1), (12), (13), and (37) into (38), and using Lemma 1, we get

$$\psi_i(\boldsymbol{w}^*, \boldsymbol{z}^*) \leq \psi_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}^*), (\boldsymbol{c}_i, \boldsymbol{z}_{-i}^*)) = 0. \tag{39}$$

This contradicts the supposition that $\psi_i(\boldsymbol{w}^*, \boldsymbol{z}^*) = 1$ and completes the proof. ■

C. Proof of Theorem 1

Let $\boldsymbol{m}^*$ be a NE. Using (12), (13), and Lemma 2, we obtain

$$\sum_{i \in \mathcal{N}} t_i(\boldsymbol{m}^*) = \sum_{i \in \mathcal{N}} \sum_{l \in \hat{R}_i} t_i^l(\boldsymbol{m}^*) = \sum_{l \in \hat{L}} \sum_{i \in S_l} t_i^l(\boldsymbol{m}^*) = \sum_{l \in \hat{L}} \sum_{i \in S_l} \left[P_{-i}^{l*} \left(\sum_{k: l \in R_i^k} y_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*) - \frac{1}{|S_l|} c_l \right) + (p_i^{l*} - \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*))^2 \right], \tag{40}$$

where $\hat{L}$ is the set of all competitive links of the network. Applying Lemma 1 to (40), we get

$$\sum_{i \in \mathcal{N}} t_i(\boldsymbol{m}^*) = \sum_{l \in \hat{L}} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \left(\sum_{i \in S_l} \sum_{k: l \in R_i^k} y_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*) - c_l \right). \tag{41}$$

Furthermore, the KKT conditions for problem **OUT** give

$$\lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \left(\sum_{i \in S_l} \sum_{k: l \in R_i^k} y_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*) - c_l \right) = 0 \quad \forall l \in \hat{L}. \tag{42}$$

Substituting (42) in (41) completes the proof. ■

D. Proof of Lemma 3

Consider a fixed pair $(\boldsymbol{w}_{-i}, \boldsymbol{z}_{-i})$ and a fixed $\boldsymbol{x}_i = (x_i^1, \ldots, x_i^{K_i})$ such that $\sum_{k: l \in R_i^k} x_i^k \leq c_l$, and $x_i^k \geq 0$, for all $k = 1, \ldots, K_i$. First, we want to prove that there exists at least one pair $(\boldsymbol{w}_i, \boldsymbol{z}_i)$ such that $y_i((\boldsymbol{w}_i, \boldsymbol{w}_{-i}), (\boldsymbol{z}_i, \boldsymbol{z}_{-i})) = \boldsymbol{x}_i$. For that matter, we consider the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{x}_{-i}} \quad & \sum_{j \neq i} \sum_{k=1}^{K_j} w_j^k f(x_j^k) \\ \text{s.t.} \quad & \sum_{\substack{j \in S_l,\\ j \neq i}} \sum_{\substack{k:\\ l \in R_j^k}} x_j^k \leq c_l - \sum_{k: l \in R_i^k} x_i^k \quad \forall l \in L : \exists j \neq i, j \in S_l \\ & 0 \leq x_j^k \leq z_j^k \quad \forall j \neq i \quad \forall k \in \{1, \ldots, K_j\}, \end{aligned} \tag{43}$$

which maximizes the sum of surrogate functions of agents except i, when the transmission rate vector x_i is reserved for agent i.

Denote by $\bar{\boldsymbol{x}}_{-i}$ an optimal solution of problem (43) and by $(\bar{\lambda}, \bar{\gamma}, \bar{\theta})$ a set of Lagrange multipliers which along with $\bar{\boldsymbol{x}}_{-i}$ satisfy the KKT conditions of problem (43); in the set $(\bar{\lambda}, \bar{\gamma}, \bar{\theta})$, $\bar{\lambda} = (\bar{\lambda}_l : l \in L : \exists j \neq i, j \in S_l)$ is the Lagrange multiplier vector corresponding to the capacity constraints, and $\bar{\gamma} = (\gamma_j^k, j \neq i, k \in \{1, \ldots, K_j\})$ and $\bar{\theta} = (\theta_j^k, j \neq i, k \in \{1, \ldots, K_j\})$ denote the Lagrange multipliers corresponding to the nonnegativity and maximum-demand constraints, respectively.

By some algebra, we can show that if $w_i^k \geq \frac{\sum_{l \in \hat{R}_i} \bar{\lambda}_l}{f'(x_i^k)}$ and $z_i^k = x_i^k$ for all $k = 1, \ldots, K_i$, then there exists a set of Lagrange multipliers along which $(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_{-i})$ satisfies the KKT conditions of problem **OUT**; hence, $y_i((\boldsymbol{w}_i, \boldsymbol{w}_{-i}), (\boldsymbol{z}_i, \boldsymbol{z}_{-i})) = x_i$. This proves the nonemptiness of the set $A = \{(\boldsymbol{w}_i, \boldsymbol{z}_i) : y_i((\boldsymbol{w}_i, \boldsymbol{w}_{-i}), (\boldsymbol{z}_i, \boldsymbol{z}_{-i})) = \boldsymbol{x}_i\}$.

Second, we show that A has at least one member with $\psi_i((\boldsymbol{w}_i, \boldsymbol{w}_{-i}), (\boldsymbol{z}_i, \boldsymbol{z}_{-i})) = 0$. If there exists a pair $(\boldsymbol{w}_i', \boldsymbol{z}_i') \in A$ such that $\boldsymbol{z}_i' = \boldsymbol{c}_i$, then $\psi_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}), (\boldsymbol{z}_i', \boldsymbol{z}_{-i})) = 0$, and we are done. Otherwise, since there exists no weight vector $\boldsymbol{w}_i' > 0$ that can provide rate $\boldsymbol{x}_i$ for agent i when she demanded for maximum feasible rates, according to the definition of function $\psi_i(.)$ (15), $\psi_i((w_i, \boldsymbol{w}_{-i}), (z_i, \boldsymbol{z}_{-i})) = 0$, for all $(\boldsymbol{w}_i', \boldsymbol{z}_i') \in A$. Thus, in both cases, there is at least one pair $(\boldsymbol{w}_i, \boldsymbol{z}_i)$ such that $y_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}), (\boldsymbol{z}_i', \boldsymbol{z}_{-i})) = \boldsymbol{x}_i$ and $\psi_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}), (\boldsymbol{z}_i', \boldsymbol{z}_{-i})) = 0$. Now, we can set $p_i^l = \lambda_l(\boldsymbol{w}', \boldsymbol{z}')$ to complete the proof of Lemma 3. ■

E. Proof of Theorem 2

According to the argument below Lemma 3, $y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)$ along with a set of Lagrange multipliers (μ_i, β_i) satisfies the KKT conditions of problem $\textbf{NASH}_i$, where $(\mu_i = (\mu_{il} : l \in L)$ is the Lagrange multiplier vector corresponding to the capacity constraints, and $\beta_i = (\beta_i^k, k = 1, \ldots, K_i))$ is the Lagrange multiplier vector corresponding to the nonnegativity constraints. We utilize these Lagrange multiplier vectors to construct a set of Lagrange multipliers $(\eta = (\eta_l : l \in L), \alpha = (\alpha_i^k, i \in \mathcal{N}, k = 1, \ldots, K_i))$, which along with $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*)$ satisfy the KKT conditions of problem **FR**, as follows:

$$\eta_l = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) + \sum_{j\in\mathcal{N}} \mu_{jl}$$
$$\alpha_i^k = \beta_i^k + \sum_{l\in R_i^k} \sum_{j\in\mathcal{N}, j\neq i} \mu_{jl}. \tag{44}$$

Then, $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*)$ along with (η, α) satisfies the KKT conditions of problem **FR**; therefore, $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*)$ is equal to the unique optimal solution of problem **FR**, $\boldsymbol{x}^*$. ■

F. Proof of Theorem 3

The proof is provided in the paragraph above the statement of Theorem 3. ■

G. Proof of Theorem 4

We show that message profile $\boldsymbol{m}^* = (m_i^* : i \in \mathcal{N})$ defined by (24)–(25), is a NE of the game induced by the DSSO mechanism. Since $\boldsymbol{x}^*$ is the optimal solution of problem **FR**, it satisfies the KKT conditions of problem **FR** along with a Lagrange multiplier vector $(\eta = (\eta_l : l \in L), \alpha = (\alpha_i^k, i \in \mathcal{N}, k = 1, \ldots, K_i))$. Based on this, we can show that $\boldsymbol{x}^*$ along with the Lagrange multiplier vector $(\lambda = (\lambda_l : l \in L), \gamma = (\gamma_i^k, i \in \mathcal{N}, k = 1, \ldots, K_i), \theta = (\theta_i^k, i \in \mathcal{N}, k = 1, \ldots, K_i))$, where

$$\lambda_l = \eta_l \quad \forall l \in L$$
$$\gamma_i^k = \alpha_i^k \quad \forall i \in \mathcal{N} \quad \forall k = 1, \ldots, K_i$$
$$\theta_i^k = 0 \quad \forall i \in \mathcal{N} \quad \forall k = 1, \ldots, K_i, \tag{45}$$

satisfies the KKT conditions of problem **OUT** when agents announce weights $\boldsymbol{w}^*$ and maximum demands $\boldsymbol{z}^*$ defined in (24) and (25). Therefore, $\boldsymbol{x}^*$ is the optimal solution of problem **OUT** for the pair $(\boldsymbol{w}^*, \boldsymbol{z}^*)$, and we have $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*) = \boldsymbol{x}^*$. Let $(\lambda(\boldsymbol{w}^*, \boldsymbol{z}^*), \gamma(\boldsymbol{w}^*, \boldsymbol{z}^*), \theta(\boldsymbol{w}^*, \boldsymbol{z}^*))$ denote the centroid of the set of Lagrange multiplier vectors of problem **OUT**. By simple algebra we can show that for each i, $y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)$ along with Lagrange multipliers $(\mu_i = (\mu_{il} : l \in L), \beta_i = (\beta_i^k, k = 1, \ldots, K_i))$, where

$$\mu_{il} = \begin{cases} \frac{\theta_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*)}{\left|\{l' \in R_i^k : y_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*) = c_{l'}\}\right|}, & \exists k : y_i^k(\boldsymbol{w}^*, \boldsymbol{z}^*) = c_l \\ 0, & \text{otherwise} \end{cases}, \tag{46}$$

for all $l \in L$, and

$$\beta_i^k = \gamma_i^k, \tag{47}$$

for all $k = 1, \ldots, K_i$, satisfies the KKT conditions of problem $\mathbf{NASH}_i$. Therefore, $y_i(\boldsymbol{w}^*, \boldsymbol{z}^*)$ is the best transmission rate vector agent i can expect, when other agents adhere to the message profile $\boldsymbol{m}_{-i}^*$. As a result, agent i has no incentive to deviate from $\boldsymbol{m}^*$ for achieving better rates. The only incentive she could possibly have for deviation is to reduce the penalty tax she must pay. However, at message profile $\boldsymbol{m}^*$, $\psi_i(\boldsymbol{w}^*, \boldsymbol{z}^*) = 0$, and $p_i^{l*} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$, $l \in \hat{R}_i$, meaning that agent i pays no penalty tax. Therefore, for each $i \in \mathcal{N}$, message m_i^* is the best response of agent i to the message $\boldsymbol{m}_{-i}^*$ of the others, hence the message profile $\boldsymbol{m}^*$ is a NE. ■

H. Proof of Theorem 5

To prove this theorem, we use the following lemma.

Lemma 6: At any NE point $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ of the game induced by the DSSO mechanism, we have $z_i^{*k} = c_i^k$, for all $i \in \mathcal{N}$ and $k = 1, \ldots, K_i$.

Proof: We prove this lemma by contradiction. Suppose there exists a NE $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ such that $z_i^{*k} \neq c_i^k$, for a pair of i and k. Then, according to the definition of function $\psi_i(.)$ presented in (15), Lemma 2 is satisfied only if there does not exist a $\boldsymbol{w}_i' > \mathbf{0}$ such that

$$y_i((\boldsymbol{w}_i', \boldsymbol{w}_{-i}^*), (c_i^1, \ldots, c_i^k, \boldsymbol{z}_{-i}^*)) = y_i(\boldsymbol{w}^*, \boldsymbol{z}^*). \tag{48}$$

When all of the routes of agent i are fully utilized, following the definitions and approach presented in the proof of Lemma 3, we can show that the weight vector $\boldsymbol{w}_i'$ where

$$w_i'^k = \frac{\sum_{l\in\hat{R}_i^k} \bar{\lambda}_l}{f'(x_i^k)}, \tag{49}$$

for all $k = 1, \ldots, K_i$ is a strictly positive vector that satisfies condition (48). Therefore, the absence of such a strictly positive vector shows that for at least one of the routes of agent i, say route R_i^k, all of the links $l \in \hat{R}_i^k$ has some unused bandwidth (in this case, we have $\bar{\lambda}_l = 0$ for all $l \in \hat{R}_i^k$, thus $w_i'^k$ defined in (49) is zero and hence is not strictly positive). Having an underutilized route R_i^k contradicts the efficiency of the allocation stated in Theorem 2, because the manager can increase the social-welfare function by increasing the transmission rate of agent i along route R_i^k. This contradiction shows that the supposition is false, and this completes the proof. ■

Now suppose that $\boldsymbol{m}^* = (\boldsymbol{w}^*, \boldsymbol{z}^*, \boldsymbol{p}^*)$ is a NE. Then, due to Lemma 6, we have $z_i^{*k} = c_i^k$, for all $i \in \mathcal{N}$ and $k = 1, \ldots, K_i$. Furthermore, from Theorem 2, we have $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*) = \boldsymbol{x}^*$. Therefore, since $\boldsymbol{m}^*$ is a NE, $y_i(\boldsymbol{w}^*, \boldsymbol{z}^*) = x_i^*$ is an optimal solution of problem $\mathbf{NASH}_i$, for each $i \in \mathcal{N}$. Since $0 < x_i^{k*} < c_i^k$, for all i and k, the KKT conditions of problem $\mathbf{NASH}_i$ imply

$$\left.\frac{\partial V_i}{\partial x_i^k}\right|_{\boldsymbol{x}_i^*} = \sum_{l\in\hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \quad \forall i \in \mathcal{N}. \tag{50}$$

On the other hand, $\boldsymbol{y}(\boldsymbol{w}^*, \boldsymbol{z}^*) = \boldsymbol{x}^*$ is the optimal solution of problem **OUT** for given vectors $\boldsymbol{w}^*$ and $\boldsymbol{z}^*$, thus it satisfies the KKT conditions of problem **OUT**. Since $0 < x_i^{k*} < z_i^{*k} = c_i^k$, we have

$$w_i^{*k} f'(x_i^{*k}) = \sum_{l\in\hat{R}_i} \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*) \quad \forall i \in \mathcal{N}. \tag{51}$$

From (50) and (51), it follows that for each $i \in \mathcal{N}$, we have $w_i^{*k} = \frac{\partial V_i}{\partial x_i^k}|_{\boldsymbol{x}_i^*} / f'(x_i^{*k})$. Moreover, because of Lemma 1, $\boldsymbol{p}_i^*$ is also determined uniquely as $p_i^{l*} = \lambda_l(\boldsymbol{w}^*, \boldsymbol{z}^*)$, $l \in \hat{R}_i$, so the NE point is unique. ■

I. Proof of Theorem 6

The rate vector $\boldsymbol{y}(\boldsymbol{w}, \boldsymbol{z})$ is a feasible point of problem **OUT**. For any given vectors $\boldsymbol{w}$ and $\boldsymbol{z}$, the feasible set of **OUT** is a subset of the feasible set of **FR**. Thus, $\boldsymbol{y}(\boldsymbol{w},\boldsymbol{z})$ is a feasible point of problem **FR**. ■

J. Proof of Lemma 4

Let $\boldsymbol{m}=(\boldsymbol{w},\boldsymbol{z},\boldsymbol{p})$ be a fixed point of function μ_{BE}. Then, we have

$$w_i{}^k f'(y_i{}^k(\boldsymbol{w},\boldsymbol{z})) = \frac{\partial V_i}{\partial x_i^k}\Big|_{y_i(\boldsymbol{w},\boldsymbol{z})}. \quad (52)$$

Since $\boldsymbol{y}(\boldsymbol{w},\boldsymbol{z})$ satisfies the KKT conditions of problem **OUT**, using (52) and by some algebra, we can show that $\boldsymbol{y}(\boldsymbol{w},\boldsymbol{z})$ also satisfies the KKT conditions of problem **FR**, and hence is the efficient allocation, i.e.,

$$\boldsymbol{y}(\boldsymbol{w},\boldsymbol{z}) = \boldsymbol{x}^*. \quad (53)$$

Substituting (53) into (30) and comparing the result with (25) proves $\boldsymbol{w}=\boldsymbol{w}^*$ and $\boldsymbol{z}=\boldsymbol{z}^*$, and subsequently $\boldsymbol{p}=\boldsymbol{p}^*$, hence the message profile $\boldsymbol{m}^*=(\boldsymbol{w}^*,\boldsymbol{z}^*,\boldsymbol{p}^*)$ is the unique fixed point of function μ_{BE}. ■

K. Proof of Lemma 5

Under Assumption 2 the function T is continuously differentiable. The following lemma provides a sufficient condition for the continuously differentiable function T to be a contraction.

Lemma 7: A function T is a contraction mapping if $\sup_{\boldsymbol{w}} \|DT(\boldsymbol{w})\| < 1$, where $DT(\boldsymbol{w})$ is the Jacobian matrix of T and $\|.\|$ is any matrix norm.

The proof of this lemma follows from Conlisk [43]. Using the absolute row-sum norm, Lemma 7 says that function T is a contraction mapping if

$$\sum_{j\in\mathcal{N}, a\in\{1,\ldots,K_j\}} \left|\partial T_i^k(\boldsymbol{w})/\partial w_j^a\right| < 1, \quad (54)$$

at every $\boldsymbol{w}$, for each $i\in\mathcal{N}$ and each $k=1,\ldots,K_i$. Substituting the partial derivatives of function T in (54) gives

$$\frac{\left|V_i^{k''}(y_i^k(\boldsymbol{w},\boldsymbol{c}))f'(y_i^k(\boldsymbol{w},\boldsymbol{c})) - f''(y_i^k(\boldsymbol{w},\boldsymbol{c}))V_i^{k'}(y_i^k(\boldsymbol{w},\boldsymbol{c}))\right|}{(f'(y_i^k(\boldsymbol{w},\boldsymbol{c})))^2} \times \sum_{\substack{j\in\mathcal{N},\\ a\in\{1,\ldots,K_j\}}} \frac{f'(y_j^a(\boldsymbol{w},\boldsymbol{c}))\left|\det(Q_{-\sum_{b=1}^{j-1}K_b+a,\sum_{b=1}^{i-1}K_b+k}(\boldsymbol{w}))\right|}{|\det(Q(\boldsymbol{w}))|} < 1, \quad (55)$$

as a sufficient condition for function T to be a contraction, where $Q(\boldsymbol{w})$ is a block matrix of the form

$$Q(\boldsymbol{w}) = \begin{pmatrix} A(\boldsymbol{w}) & R \\ R^T & 0 \end{pmatrix}, \quad (56)$$

where

$$A(\boldsymbol{w}) = \begin{pmatrix} -w_1^1 f''(x_1^1) & & & \\ & -w_1^2 f''(x_1^2) & & \\ & & \ddots & \\ & & & -w_N^{K_N} f''(x_N^{K_N}) \end{pmatrix}, \quad (57)$$

is a $P\times P$ diagonal matrix, and for all $u,v\in\{1,\ldots,P\}$, $Q_{-u,v}(\boldsymbol{w})$ denotes matrix $Q(\boldsymbol{w})$ when row u and column v are removed.

In the following, we show that condition (55) is satisfied if the network manager chooses $f(x)=\log(x/b+1)$ as the surrogate function. Substituting the surrogate function $f(x)=\log(x/b+1)$ into the first multiplicative term of (55), we get

$$\frac{\left|V_i^{k''}(y_i^k(\boldsymbol{w},\boldsymbol{c}))f'(y_i^k(\boldsymbol{w},\boldsymbol{c})) - f''(y_i^k(\boldsymbol{w},\boldsymbol{c}))V_i^{k'}(y_i^k(\boldsymbol{w},\boldsymbol{c}))\right|}{(f'(y_i^k(\boldsymbol{w},\boldsymbol{c})))^2} = \left|V_i^{k'}(y_i^k(\boldsymbol{w},\boldsymbol{c})) + (y_i^k(\boldsymbol{w},\boldsymbol{c})+b)V_i^{k''}(y_i^k(\boldsymbol{w},\boldsymbol{c}))\right|. \quad (58)$$

Using the definition of b presented in Assumption 3, it can be shown that the right-hand side of (58) is a decreasing function of $y_i^k(\boldsymbol{w},\boldsymbol{c})$. Therefore, it is upper bounded by its value at 0, i.e.,

$$\frac{\left|V_i^{k''}(y_i^k(\boldsymbol{w},\boldsymbol{c}))f'(y_i^k(\boldsymbol{w},\boldsymbol{c})) - f''(y_i^k(\boldsymbol{w},\boldsymbol{c}))V_i^{k'}(y_i^k(\boldsymbol{w},\boldsymbol{c}))\right|}{(f'(y_i^k(\boldsymbol{w},\boldsymbol{c})))^2} \le \left|V_i^{k'}(0) + bV_i^{k''}(0)\right|. \quad (59)$$

Now we work on the second multiplicative term of (55). From the second iteration of the CBE process, the weights announced by the agents are of the form $w_i^k = \frac{\partial V_i}{\partial x_i^k}|_{y_i(\boldsymbol{w},\boldsymbol{c})}/f'(y_i^k(\boldsymbol{w},\boldsymbol{c}))$, which is simplified to

$$w_i^k = (y_i^k(\boldsymbol{w},\boldsymbol{c})+b)V_i^{k'}(y_i^k(\boldsymbol{w},\boldsymbol{c})), \quad (60)$$

for the surrogate function $f(x)=\log(x/b+1)$. According to the definition of b, the right-hand side of (60) is an increasing function of $y_i^k(\boldsymbol{w},\boldsymbol{c})$. Therefore, since $0\le y_i^k(\boldsymbol{w},\boldsymbol{c})\le c$, we have

$$bq \le bV_i^{k'}(0) \le w_i^k \le (c+b)V_i^{k'}(c) \le (c+b)Q, \quad (61)$$

for all $i\in\mathcal{N}$ and $k=1,\ldots,K_i$, where the first and last inequalities follow from the definitions of $q=\min_{i,k} V_i^{k'}(0)$ and $Q=\max_{i,k} V_i^{k'}(c)$ presented in Assumption 3. Combining (61) with the fact that $1/(c+b)^2 \le -f''(x) \le 1/b^2$ for all $x\in[0,c]$ gives the following bounds on the diagonal elements of matrix A:

$$\frac{bq}{(c+b)^2} \le -w_i^k f''(x_i^k) \le \frac{(c+b)Q}{b^2}, \quad (62)$$

where $i\in\mathcal{N}$ and $k=1,\ldots,K_i$. Using the bounds of (62) and after some algebra we can show that

$$|\det(Q(\boldsymbol{w}))| \ge \left(\frac{bq}{(c+b)^2}\right)^{P-L} \left|\det(R^T R)\right|, \quad (63)$$

and

$$|\det(Q_{-u,v}(\boldsymbol{w}))| \le \left(\frac{(c+b)Q}{b^2}\right)^{P-L-1} \left|\det(R_{-v}^T R_{-u})\right|, \quad (64)$$

for all $u,v\in\{1,\ldots,P\}$. Therefore, since $f'(x)<1/b$ for all $x\in[0,c]$, we can obtain an upper bound for the second multiplicative term of (55) as follows:

$$\sum_{\substack{j\in\mathcal{N},\\ a\in\{1,\ldots,K_j\}}} \frac{f^{'}(y_j^a(\boldsymbol{w},\boldsymbol{c}))\left|\det(Q_{-\sum_{b=1}^{j-1}K_b+a,\sum_{b=1}^{i-1}K_b+k}(\boldsymbol{w}))\right|}{|\det(Q(\boldsymbol{w}))|}$$

$$\leq \left(\frac{c+b}{b}\right)^{3(P-L)-1}\frac{Q^{P-L-1}}{q^{P-L}}\frac{\sum_{u=1}^{P}\left|\det(R^T_{-\sum_{b=1}^{i-1}K_b+k}R_{-u})\right|}{|\det(R^TR)|}$$

$$\leq \left(\frac{c+b}{b}\right)^{3(P-L)-1}\frac{Q^{P-L-1}}{q^{P-L}}\frac{1}{S}, \tag{65}$$

where the last inequality follows from the definition of the network index S (27).

Substituting (59) and (65) into the left-hand side of (55), and using Assumption 3 proves the fulfillment of condition (55) and hence contractiveness of function T. This completes the proof of Lemma 5. ■

L. Proof of Theorem 7

According to Lemma 5 choosing an appropriate surrogate function $f(.)$ makes the update rule T, from the second iteration on, a contraction mapping. This proves the global convergence of the CBE process, as the Banach fixed point theorem states that a tatonnement process is globally convergent if its update rule is a contraction. The convergence of the BE and CBE processes are equivalent. Therefore, the BE process always converges to its unique fixed point which by Lemma 4 is a NE of the game induced by the DSSO mechanism (see Lemma 4). Therefore, the DSSO mechanism is stable under the BE process, hence it is dynamically stable. ■

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