An Efficient Market Design for Electricity Networks with Strategic Users Possessing Local Information*

Mohammad Rasouli**, and Demosthenis Teneketzis+,
**Management Science and Engineering Department, Stanford, CA, 94305, USA
+Department of EECS, University of Michigan, Ann Arbor, MI 48109, USA
E-mail: rasouli,teneket@umich.edu, Tel: (734) 219-2671, (734) 763-0598

Abstract— **We consider electricity wholesale markets with multiple strategic users that possess localized information about the electricity network and can be either producers or consumers. The objective is to design a mechanism that maximizes the social welfare (the sum of the users' utilities) and has the following additional features. It satisfies the users' informational constraints along with the constraints imposed by the lines' thermal capacity limit and the network's physical laws (Kirchhoff's laws); furthermore it is budget balanced, individually rational, and price efficient. Using ideas from the theory of local public goods and auctions, we construct a social welfare maximizing mechanism that possesses all of the above features at equilibrium. We present an intuitive interpretation of the mechanism and discuss possible extensions of the model considered in the paper.**

I. Introduction

The electricity industry has been traditionally regulated by government to run under a predetermined-price cost-minimizing monopoly. In past decades, there has been world-wide tendency for restructuring the industry toward a free competitive market [1]. As a result of restructuring, different sectors of the industry, i.e. generation, transmission and distribution (retailers), have been separated and are run by different entities (vertical disintegration). Moreover, in the generation sector wholesale markets (e.g spot and day-ahead markets) with multiple generators (producers) have been introduced (horizontal disintegration), thus resulting in an oligopoly of generators in the wholesale market instead of a monopoly. There have been different practices of electricity restructuring from California and Pennsylvania-New Jersey-Maryland (PJM) to British and Australian markets.

Designing an efficient wholesale market in the restructured electricity industry is a formidable task for the following reasons. i) As a trading commodity, electricity has unique features [2]. In particular, network flows are interconnected through Kirchhoff's laws (KVL and KCL). Furthermore, the flows are limited by thermal capacity constraints of the lines. Kirchoff's laws introduce a loop flow effect which, combined with the lines' thermal capacity constraints, couple the producers' dispatch and limit the set of feasible dispatches to a non-convex set. Therefore, loop flow effect ties the flow of power in each line to all the other flows in the network. Moreover, the network interconnection among producers results in externalities both negative (dispatch in the same direction) and positive (dispatch in the opposite directions). As a result, production of electricity at each node is tied to production at all other nodes. Consequently a bilateral contract between a generation company and demand will affect the possible dispatch of electricity at all other nodes. Therefore, allocating optimal dispatch of electricity, constrained by the electricity transmission network constraints, cannot be achieved by free markets [3]. ii) The restructuring of the electricity industry resulted in an oligopoly of strategic producers with market power. Generation companies can manipulate the market so as to increase their own profit at the cost of reducing the social welfare. Market power was exercised by sellers even in situations where they had less than 10% market share (see [4], [5]). The structural flaws of electricity markets under deregulation have added to the potential for market power and gaming [6]. The specific features of electricity markets which give rise to gaming are lack of storage, inelasticity (or lack of elasticity) of demand[1], and, potentially, the network's structure. Electricity cannot be stored commercially. Therefore, any change in the demand for electricity from the predicted amount must be met in real-time. This fact combined with the inelasticity of demand along with limits on supply (that is constrained by the generation capacity) give generation companies the power to manipulate the market, and result in very high prices in peak demand [9], [10]. Network constraints can potentially give market power to producers for the following reasons. Due to transmission limits and congested lines, part of the network may become isolated from rest of the network. This results in market power for small local producers. Technological developments such as smart grids (that introduce demand

*This research was supported in part by NSF Grant CNS-1238962 and ARO grant W911NF-17-1-0232. The authors would like to thank William Hogan, Paul Milgrom, Robert Wilson, Asuman Ozdaglar, Ian Hiskens, Mohammad Akbarpour, and Shaowei Ke for useful discussions.
** Corresponding Author.

[1]Inelastic demand has been observed in real-time markets and congested day-ahead markets (see [7], [8]).

response and elastic demand to the wholesale market) can help in overcoming some of the structural flaws of electricity markets.

The question of interest in this paper is how to design markets for efficient dispatch of electricity considering all the above challenges. We aim at designing markets that are: 1) budget balanced (the payments by all users add up to zero); 2) individually rational (users voluntarily participate in the production/consumption process because their utility at equilibrium is not less than their utility from non-participation); 3) price efficient (the price a producer is paid, or respectively a consumer pays, per unit of power produced, or respectively consumed, is equal to sum of his marginal cost of production, respectively his marginal utility of consumption, the saturation price for his limited production capacity, respectively his limited consumption capacity, and the congestion price of the line to which he is connected.); and 4) social welfare maximizing. We also consider information constraints of the users by assuming that each user has only local knowledge about the topology of the network and the characteristics of the lines that connect him with his immediate neighbors.

A. Existing Approaches to the Design of Electricity Markets with Network Constraints

Currently, there are two approaches that incorporate network constraints in the design of electricity markets [11]. These two approaches use two different set of prices and they are: (1) the integrated market design that uses nodal prices (one price for each node in the network); and (2) the coordinated market design that uses prices for allocating rights (one price for transmission rights between any two nodes) and prices for allocating electricity dispatch (one price for each node in the network).

The most common practice of integrated market design (nodal pricing) is Supply Function (SF) auctions [12]–[14]. In these auctions, producers bid their entire cost function to the system operator; the system operator then calculates the most economically efficient dispatch (that is, he clears the market by dispatching minimum cost first [15], [16]), subject to network constraints. The price paid to each producer is the marginal cost of production at the node the producer is located in. SF auctions are used to clear multiple possible demands due to demand uncertainty or demand variation during the day (see [12]–[14]). In general, SF auctions are budget balanced and individually rational, but they have a range of equilibria which do not implement the social welfare maximizing outcome and they are not necessarily price efficient [15]. The inefficiency of SF auctions has been studied further within the context of polynomial supply functions [17], oligopoly models with capacity constraints [18], [19], models with a pivotal bidder [20], models with limited number of price bids [21], models with demand uncertainty [22], and, within the context of duopoly, models with finite piece-wise linear cost function [23]. A review of the SF auctions literature is provided in [24].

Cournot auctions have been used to approximate SF electricity auctions [12]. These auctions are price efficient as the price at all Nash equilibria is equal to marginal cost of production, they are budget balance, and they are individually rational, but they do not achieve social welfare maximizing allocations at Nash equilibria [13], [14].

For coordinated market design, first the transmission rights and next the electricity dispatches are allocated. Transmission rights are allocated to all producers simultaneously using a pooling auction; this is because transmission rights cannot be traded bilaterally and independently as they are interconnected because of the loop flow effect that is due to Kirchhoff's current and voltage laws (KCL, KVL). Coordinated market design is proposed in the form of physical transmission right (PTR), financial transmission right (FTR) and flow gate right (FGR) [25]. In PTR, the physical capacity of a line is allocated to generation companies to use it in order to send electricity to other nodes in the network. In FTR only the financial right of the lines is allocated; the owner is paid for his amount of FTR according to the difference in the price of electricity at the two ends of the line. Similar to FTR, FGR does not allocate physical rights, but the price paid to the holder is the marginal improvement in social welfare due to the increase in the capacity of the line. FGR is the correct signal for investment to increase the capacity of the line.

In competitive markets with full information, both the integrated market design and the coordinated market design lead to social welfare maximizing outcome (See [26]- [27] for integrated, and [28] for coordinated). When market power exists (even under full –information) the current design of these markets is not efficient [29] and the comparison between the performance of the two approaches is not completely known [11]. The problem becomes even more challenging when a producer owns multiple electricity generation firms located at different nodes, or when there are multiple producers located at a single node [27]. Similarly, comparison among different approaches to coordinated markets is not clear [30]. Holders of the physical right can choose not to use them so as to increase the price difference at the two ends of the line, while holders of the financial right have incentive to under-produce electricity at the congested nodes so as to increase price in those nodes and collect more payment. Rules such as use-it-or-loose-it or use-it-or-sell-it are introduced to reduce such market manipulations.

The above discussion demonstrates the necessity for designing efficient electricity wholesale markets with network constraints which is the topic of this paper.

Our approach to designing efficient electricity wholesale markets is based on mechanism design for local public goods problems. In electricity networks, the actions (e.g. power production, power consumption) taken at a network node directly influence the actions and utilities of strategic users at that node and the immediate neighborhood of the node (due to the physical laws of power flows), thus, the design of electricity markets is a local public goods problem.

B. Existing Results for Mechanism Design

There exists a large literature on local public goods within the context of local public good provisioning by various municipalities that follows seminal work of [31]. All this literature considers network problems in which strategic agents/individuals decide where to locate; the choice of their location is based on their information about the revenue and expenditure patterns (on local public goods) of various municipalities. In contrast to the above literature, in this paper we address the problem of determining the optimal levels of local public goods for a *given* network. Therefore, our problem is distinctly different from those considered in the literature following [31]. Recently, [32] and [33] investigated the influence of selfish/strategic users' behavior on the provision of local public goods in networks with fixed links among users. They analyzed Nash Equilibria (NE) of the game where the users' utilities are linear and each user's strategy is to choose the level of effort that maximizes his own payoff from the provisioning of the local public good. It was shown in [32] and [33] that none of the NE of the aforementioned games result in provisioning of local public goods that is social welfare maximizing; thus, the work presented in [32] and [33] does not achieve our goals (stated in Section II).

In this paper, we consider a network model that is more general than the model of [32] and [33]. Specifically, we consider a fixed network where the actions taken at each node n affect the utilities of all the strategic users in node n and all the nodes that are directly connected to n. Furthermore, each user's utility is quasilinear (convex plus linear, in contrast to [32]- [33] where the agents' utilities are linear) and is his own private information. The constraints due to the physical laws governing electricity networks are present in our model but not in [32] and [33]. Our approach to the local public goods problem is different than that of [32] and [33]; it is based on mechanism design/implementation theory. The objectives of our paper are different from those of [32] and [33].

Prior work on mechanism design for public goods (not local public goods) has appeared in [34]–[37]. Our work is inspired by [35]. In [35], Hurwicz presents a mechanism that implements in NE the Lindahl correspondence (social welfare correspondence). Our problem is different from Hurwicz's in two key aspects: (1) our model includes the physical constraints imposed by Kirchoff's laws; such constraints are unique to electricity networks and are not present in Hurwicz's model (as well as in [34], [34], [36], [37]). As a result, the mechanism design problem is distinctly different and much more challenging than that of [35] (as well as those of [34], [34]–[37]). (2) We consider a local public goods problem as opposed to a public goods problem (that is studied in [35]). As a result, Hurwicz's ideas and methodology can not be directly used to obtain our objectives (stated in Section II) even in the absence of the physical constraints imposed by Kirchoff's laws.

Prior work on mechanism design for local public goods can be found in [38]. The key difference between [38] and our paper is in the model. The network in [38] investigates local public goods problems motivated primarily by power allocation in wireless networks, thus the model of [38] does not include constraints due to Kirchoff's laws which, as stated above, are unique to electricity networks. Such constraints result in a problem that is distinctly different from and more difficult than that of [38].

C. Contribution

We design a mechanism that has the following features: It satisfies the strategic producers' and consumers' informational constraints, the network's physical laws (Kirchhoff's laws) and the lines' thermal capacity constraints. Furthermore, it is social welfare maximizing, budget-balanced at equilibrium, individually rational, and price-efficient.

To the best of our knowledge, a mechanism possessing all the above desirable features does not currently exist in the literature.

D. Organization

The rest of the paper is organized as follows. We present the network model and our objective in Section II. Then, in Section (III), we discuss how the electricity network's features affect the design of the market. In Section IV, we present our market design and interpret it. We establish its efficiency, and we compare it with the mechanisms currently used in electricity markets. In Section V, we extend the results to sparse networks. We conclude in Section VI where we present some open problems associated with the proposed mechanism.

II. Model and Objective

A. The Model

We consider an electricity network with K interconnected nodes, N strategic users located in these nodes, and one network manager, called the independent system operator (ISO). We denote by $\mathcal{K} := \{1, 2, ..., K\}$ the set of the network's nodes and by $\mathcal{N} := \{1, 2, ..., N\}$ the set of the network's users. The neighborhood of node k, denoted by R_k, is the set of nodes that are directly connected to node k including node k itself. Each user may be both producer or consumer. We denote by e_n, $n \in N$, user n's energy consumption/production. If $e_n > 0$ (respectively, $e_n < 0$) user n is a producer (respectively a consumer). We assume that for $n \in \mathcal{N}$, $-w_n \leq e_n \leq x_n$ where x_n is user n's production capacity and $|w_n|$ is his maximum consumption capacity. User n's aggregate utility function is given by $u_n(e_n) - t_n$, where $u_n(e_n)$ describes his cost from producing/consuming energy e_n and t_n denotes his tax/subsidy as a result of the production/consumption process taking place in the network. The cost functions $u_n : \mathbb{R} \to \mathbb{R}$, $n \in \mathcal{N}$, are strictly convex and increasing with $u_n(0) = 0$, and $t_n \in \mathbb{R}$. The set of users in node k is denoted by $\mathcal{N}_k$. In this paper we initially assume that $\mathcal{N}_k = 1$, later in Section V we consider sparse networks where $\mathcal{N}_k \leq 1$. The operating angle of node k is denoted by θ_k. There is one slack bus/slack node denoted by n_0 the operating angle of which is $\theta_{n_0} = 0$. The operating angles of the remaining nodes are measures with respect to θ_{n_0}. The flow of power from node k to node j is determined by a function $f(\theta_k - \theta_j)$ and is denoted by I_{kj}. In this paper we consider a second-order approximation of the power flow I_{kj} that is given by

$$I_{kj} = B_{kj} \times (\theta_k - \theta_j) + \frac{1}{2} G_{kj}(\theta_k - \theta_j)^2, \qquad (1)$$

where B_{kj} (respectively, G_{kj}) is that first derivative (respectively, the second derivative) of $f_{kj}(.)$ with respect to its argument, evaluated at zero. In this approximation, the second-order term captures the loss of power along the line from node k to node j. The total loss in the line connecting nodes k and j is $I_{kj} + I_{jk} = G_{kj}(\theta_k - \theta_j)^2$. The power flow I_{kj} can not exceed the thermal capacity of the line connecting node k and j. We denote this thermal capacity by L_{kj}, and we must have $I_{kj} \leq L_{kj}$ for all $k, j \in K$.

The information structure of the model (who knows what) is as follows. Each user n knows R_n and his cost, $u_n \in U$, where U is the space of function $u : \mathcal{R} \rightarrow \mathcal{R}$ that are strictly convex and increasing with $u(0) = 0$. The space U is common knowledge among all users, whereas u_n is known only by user $n, n \in N$. We assume a non-Bayesian environment, that is, we assume no (prior) pdf on U. Only the users who are in R_{n_0} know that n_0 is the slack node (i.e. they know $\theta_{n_0} = 0$); the users who are not in R_{n_0} only know that there exists a slack node but do not know which node it is. The operating angles θ_k of nodes $k \neq n_0$ are measured with respect to θ_{n_0}. A user located in node k knows B_{kj} and G_{kj} for all $j \in R_k$, that is, user n, located in node k, knows the precise structure of the local network that connects him with his neighbors in R_n. The ISO knows the topology of the whole network along with all the parameters B_{kj}, G_{kj}, $k \in N, j \in N$, but does not know the users' utilities.

The above discussion completes the specification of the network model along with the description of the information available to the strategic users and the ISO.

Remark II.1. *We use the second-order approximation model because: (1) We can construct an efficient mechanism for this model; and (2) the performance achieved using this model is very close to that achieved by the full AC model (see Section VI).*

B. Objective

Our objective is to design a set of rules (a *mechanism*) according to which the strategic users (consumers and producers) and the ISO interact through the electricity network. The mechanism must take into account the network's information structure and the network constraints (the lines' thermal constraints), its physical laws (Kirchoff's laws), and must possess certain properties which we will state below.

A mechanism consists of two components:

(1) A message/strategy space $M = M_1 \times M_2 \times ... \times M_N$, that is a communication alphabet through which the strategic users send information to the ISO; M_i denotes the message space/communication alphabet of user $i, i \in N$.

(2) An outcome function that determines the power generation/power consumption e, $e := (e_1, e_2, ..., e_N)$ along with monetary incentives t, $t := (t_1, t_2, ..., t_N)$, that are taxes or subsidies provided to the users after the communication/message exchange process between the strategic users and the ISO terminates. The power generation/power consumption profile e and the tax/subsidy profile t are functions of the users' terminal/final messages to the ISO. Such a mechanism induces a game among the strategic users. We consider Nash Equilibrium (NE) as the solution concept of this game. At the end of this section we discuss why NE is an appropriate solution concept for our problem.

We want to design a mechanism that possesses the following properties:

(P1) Budget Balance: Let $t_i(m^*)$ denote the payment to user i, $i = 1, 2, ..., N$ at a NE $m^* := (m_1^*, m_2^*, ..., m_N^*)$ of the game induced by the mechanism. We must have $\sum_{i=1}^N t_i(m^*) = 0$ for all $m^* \in M^*$, where M^* is the set of NE of the game induced by the mechanism.

(P2) Individual Rationality: Let $u_i(e_i^*(m^*)) - t_i(m^*)$ denote the aggregate cost of user i, $i \in N$, at a NE of the game induced by the mechanism. We must have $u_i(e_i(m^*)) - t_i(m^*) \leq 0$ for all $m^* \in M^*$, for all $i \in N$, where 0 is the aggregate cost of user i when he decides not to participate in the production/consumption process (Note that non-positive aggregate cost implies non- negative aggregate utility/reward).

production/power consumption of agent $i, i \in N$, corresponding to NE $m^* \in M^*$, and let $e(m^*) := (e_1(m^*), ..., e_N(m^*))$ denote the power production/power consumption profile corresponding to $m^* \in M^*$. We must have $e(m^*) = \hat{e}$, $\hat{e} := (\hat{e}_1, \hat{e}_2, ..., \hat{e}_N)$ for all $m^* \in M^*$, where $\hat{e}$ is the solution of the centralized optimization problem (Socially Optimal I) presented in Section II-C below.

(P4) Price Efficiency: The price $p_i(m^*)$ producer i is paid (respectively consumer i pays) per unit of power produced (respectively, consumed) at any NE $m^* \in M^*$ be equal to the sum of his marginal cost of production (respectively, his marginal utility of consumption), the saturation price for his limited production capacity (respectively, his limited consumption capacity), and the congestion price of the lines to which he is connected.

We call any mechanism that possess properties (P1)-(P4) *efficient*. Thus, our objective is to design an efficient mechanism for the electricity network model described in Section II-A.

Remark II.2. *NE is the solution concept/equilibrium concept used in most studies of electricity networks with strategic users. (See [11], [39], [40] and references therein). The above-mentioned papers consider electricity markets with strategic users and symmetric information. They ar-*

gue that by eliminating information asymmetries and their corresponding information rent one can develop a basic understanding of electricity market problems and develop rigorous analytical results. For example, [40] argues that symmetric information of the bidders (GenCos) is a common rationale and a reasonable assumption for oligopolistic electricity markets, because GenCos monitor perfectly each others' technology and capacity. In our work, we assume that users (GenCos) have asymmetric information (even if they can monitor perfectly each others' technology and capacity, they do not know each others' valuation/utilities), thus the game induced by the mechanism is one of asymmetric information where the environment is non-Bayesian. For this situation NE is still an appropriate solution concept because according to Nash (Nash's PhD thesis [41], pages 21-24), NE can be interpreted in two ways: 1. As the solution concept/outcome of a game of complete/symmetric information when all agents are rational. 2. As the result of a game of asymmetric information where strategic agents are involved in an unspecified message exchange process in which they grope their way to a stationary message and in which the Nash property (NE) is a necessary condition for stationarity. This is the so-called "mass-action" interpretation of NE (See also [42] page 644). It is the second interpretation of NE that we adopt in this paper.

We conclude this section on modeling and objectives by presenting below the centralized optimization problem "Socially Optimal I" that will be useful to the discovery of the properties of the mechanism we present in Section IV.

C. Problem Social Optimal I

The socially optimal (centralized) electricity dispatch problem is

$$\min_{\theta_n, n\in\mathcal{N}} \sum_{n\in N} u_n(e_n(\theta_{R_n})) \qquad \text{(Socially Optimal I)}$$

$$s.t. \quad -w_n \leq e_n \leq x_n \quad \forall n \in N \tag{2}$$

$$I_{kj} \leq L_{kj} \quad \forall k, j \in K \tag{3}$$

where

$$\theta_{R_n} := (\theta_k, k \in R_n) \tag{4}$$

$$I_{kj} = B_{kj} \times (\theta_k - \theta_j) + \frac{1}{2} G_{kj} (\theta_k - \theta_j)^2 \quad \forall k, j \in K \tag{5}$$

$$e_n(\theta_{R_n}) = \sum_{k\in R_n} I_{nk} \tag{6}$$

III. Effect of Network on Electricity Markets

The electricity network model in Section II-A has the following features that affect the design of the market.

First, the lines have limited capacity. The capacity of a line limits the trade between its ends. If a line is congested, then the markets on the two sides of the line are not connected, instead they are two separate local markets. The producers located on each side have local power and the markets have separate prices.

Second, the flow of electricity is subject to loop flow constraints (KVL and KCL). These constraints make electricity networks unique and different from any other network; they have complicated effects as they couple all flows and productions in the network to each other. As a result of the this coupling of productions due to the Kirchhoff's laws, a producer and a consumer cannot do a separate bilateral trade independent of the others. Instead, all trades should be cleared at the same time and in a pooling market. Thus, the network becomes a public good whose use should be determined by all the producers and consumers together.

Third, the lines have loss in delivery of electricity power. This loss is a function of the total power flow through the line. The power delivered at the one end of the line is the total power injected at the opposite end minus this loss. In markets, this loss should be shared between the buyer and the seller and this is reflected in the electricity prices. For example, consider a simple network of one producer and one consumer connected with a lossy line. If the trading commodity is electricity at the producer's node, then the consumer should endogenize the cost of loss in his bids. On the other hand, if the trading commodity is electricity at the consumer's node then the producer should endogenize the cost of loss in her bids. Alternatively, the producer and consumer can trade electricity at some point in the line between them. In this case they do loss sharing. Based on this simple example, one can observe that the social welfare maximizing solution is the same for all cases but the payments of producers and consumers depend on the location where the trade takes place.

Fourth, multiple producers are located at the same node. As a result, even if the system operator determines the outflow of electricity power from every node, each user's share in that node should be determined. This may require local markets among the users at each node.

Finally, demand consists of multiple consumers at different nodes. Multiple consumers can be separated by the network constraints. Therefore, they may each have a separate market clearing price for buying electricity. This necessitates markets to discover these separate prices.

IV. Market Design for Electricity Trade over the Network: One User Per Node

We consider the model of Section II, that includes some of the features presented in Section III, namely strategic production/demand, exactly one user at each node ($\mathcal{N}_k = 1$, $\forall k \in \mathcal{N}$), second order approximation of the flows (which considers losses), and we assume that each node is connected to at least two other nodes ($|R_n| \geq 3$). This assumption is required for technical purposes; specifically, the mechanism we present in this section is efficient when $|R_n| \geq 3$, $\forall n \in \mathcal{N}$. In Section V, we extend the model to the case where there is a maximum of one user in each node.

Note that I_{nk} (given by Eq. (5)) is a convex function of θ_n and θ_k, and e_n is convex functions of θ_{R_n}. The cost for producing/consuming electricity at node n is $u_n(e_n(\theta_{R_n}))$. We assume that u_n is a strictly convex and increasing function of e_n with $u_n(0) = 0$ (this is a standard assumption in the literature). Since e_n is a convex function of θ_{R_n}, u_n is also a strictly convex function of θ_{R_n} (See [43], Theorem 5.1).

For this model, we present an efficient local public good mechanism. As noted in Section II-B, a mechanism consists of two components, a message space and an outcome function that are defined below.

Message Space The message space of the mechanism is $M = M_1 \times M_2 \times .... \times M_N$, where M_n is the message space of agent n, $n \in \mathcal{N}$,

$$M_n = [0, 2\pi]^{|R_n|} \times \mathbb{R}_+^{|R_n|}, \tag{7}$$

where $|R_n|$ denotes the cardinality of R_n. A message $m_n \in M_n$ is defined by

$$m_n = (\hat{\theta}^n_{R_n}, p^n_{R_n}) \tag{8}$$

where

$$\hat{\theta}^n_{R_n} := (\hat{\theta}^n_1, \hat{\theta}^n_2, ..., \hat{\theta}^n_{|R_n|}), \tag{9}$$

$$p^n_{R_n} := (p^n_1, p^n_2, ..., p^n_{|R_n|}), \tag{10}$$

and $1, 2, 3, ..., |R_n|$ is an (arbitrary) indexing of the users in R_n (this indexing is done for all sets R_k, $k \in \mathcal{N}$), and $\hat{\theta}^n_k$ (respectively $p^{\mathcal{N}}_k$) is the angle (respectively the price per unit of angle) proposed by user n for user $k \in R_n$. If node $n_0 \in R_n$, then $\hat{\theta}^n_{n_0} = 0$. The messages $(m_n, n \in \mathcal{N})$ are communicated to the ISO.

Outcome Function The outcome function h is defined by $h : M_1 \times M_2 \times ... \times M_n \to [0, 2\pi]^N \times \mathbb{R}^N$, where for every $m := (m_1, m_2, ..., m_N) \in M_1 \times M_2 \times ... \times M_N$,

$$h(m) = ((\theta_n(m_{R_n}), t_n((m_{R_k}), k \in R_n)), n \in \mathcal{N}), \tag{11}$$

$$m_{R_n} = (m_k, k \in R_n), \tag{12}$$

$$\theta(m_{R_n}) = \frac{1}{|R_n|} \sum_{k \in R_n} \hat{\theta}^k_n, \tag{13}$$

$$t_n((m_{R_k}), k \in R_n) = \sum_{k \in R_n} l_{n,k}(m_{R_k}) \theta_k(m_{R_k}) + \sum_{k \in R_n} p^n_k (\hat{\theta}^n_k - \hat{\theta}^{y_{n,k}+1}_k)^2, \tag{14}$$

$$l_{nk}((m_{R_k})) = (p^{y_{n,k}+1}_k - p^{y_{n,k}+2}_k), \tag{15}$$

and $y_{n,k}$ refers to the index of user n in R_k ($y_{n,k} \neq 0$ whenever $n \in R_k$, $y_{n,k} = 0$ when $n \neq R_k$). We note hat if $y_{n,k} = |R_k|$ then

$$y_{n,k} + 1 = 1, \quad y_{n,k} + 2 = 2. \tag{16}$$

A. Interpretation of the Mechanism

As pointed out in the introduction, Kirchhoff's laws couple the users' production, therefore users must coordinate their generation/consumption. This consideration results the message space proposed for the mechanism.

The outcome function of the proposed mechanism achieves two goals: (i) it eliminates the strategic users' market power; (ii) if incentivizes the strategic users to coordinate their production/consumption strategies. This is done as follows. The money each user receives/pays consists of two terms. The first term represents the payment a user gets/makes as a result of his production/consumption that depends on $\theta^n_{R_n}$). Note that the prices $\{l_{nk}, k \in R_n\}$ according to which user n gets paid per unit of angle (or pays per unit of angle) do not depend on his own message. This eliminates the agent's market power. The prices $\{l_{nk}, k \in R_n\}$ depend on the indexing of agents in $R_k, k \in R_n$. Nevertheless, we show in the next Section that the properties of the proposed mechanism are independent of this indexing. The second term incentivizes agents to coordinate their production/consumption. At equilibrium, where agents agree to coordinate their productions, this term is equal to zero.

B. Analysis of the Mechanism

To establish the properties of our mechanism, we proceed as follows. First, we write down the Karush-Kuhn-Tucker (KKT) conditions for problem (Socially Optimal I) formulated in Section II-C. Then we formulate problem (Best Response I) which determines a user's best response in the game induced by our mechanism, and write down the KKT conditions for this problem. Finally, we use the KKT conditions for problems Socially Optimal I and Best Response I to derive the properties of our mechanism.

1) KKT Conditions of for Problem Socially Optimal I: The objective function in problem (Socially Optimal I) is strictly convex in $\theta = (\theta_1, \theta_2, ..., \theta_N)$.

The domain is convex because I_{kj} is a convex function of θs and therefore the set of θs satisfying $I_{kj} \leq L_{kj}$ is convex, and furthermore, the intersection of a finite number of convex sets is also convex. Define this intersection by $Dom(\theta)$. As a result, the above optimization is a strictly convex optimization problem in θ. We present the Lagrangian and KKT conditions for this problem. These conditions will be useful in the analysis of the market design/mechanism we propose. The Lagrangian is

$$H = \sum_{n \in N} u_n(\theta_{R_n}) + \sum_{n \in N} \mu_n(x_n - e_n(\theta_{R_n})) + \sum_{n,k \in N} \lambda_{nk}(L_{nk} - I_{nk}) + \sum_{n \in \mathcal{N}} \kappa_n(e_n(\theta_{R_n}) + w_n) \tag{17}$$

Considering that

$$\frac{\partial I_{nk}}{\partial \theta_k} = -B_{nk} - G_{nk} \times (\theta_n - \theta_k) \tag{18}$$

$$\frac{\partial I_{nk}}{\partial \theta_n} = B_{nk} + G_{nk} \times (\theta_n - \theta_k) \tag{19}$$

and

$$\frac{\partial e_n(\theta_{R_n})}{\partial \theta_k} = \begin{cases} -B_{nk} - G_{nk} \times (\theta_n - \theta_k) & k \neq n, \\ \sum_{i \in R_n} B_{ni} + G_{ni} \times (\theta_n - \theta_i), & k = n, \end{cases} \tag{20}$$

the KKT conditions are

$$\frac{\partial H}{\partial \theta_n}|_{\theta_N^*, \lambda_{N\times N}^*, \mu_N^*} = [\sum_{k \in R_n} [u_k'(e_k) \frac{\partial e_k(\theta_{R_k})}{\partial \theta_n} - [\mu_k - \kappa_k] \frac{\partial e_k}{\partial \theta_n} - \lambda_{nk} \frac{\partial I_{nk}}{\partial \theta_n} - \lambda_{kn} \frac{\partial I_{kn}}{\partial \theta_n}]|_{\theta_N^*, \lambda_N^*, \mu_N^*} =$$

$$\Big[u_n'(e_n) \times [\sum_{i \in R_n} B_{ni} + G_{ni} \times (\theta_n - \theta_i)] + \sum_{k \neq n, k \in R_n} u'(e_k) \times [-B_{kn} - G_{kn} \times (\theta_k - \theta_n)] - [\mu_n - \kappa_n] \times [\sum_{i \in R_n} B_{ni} + G_{ni} \times (\theta_n - \theta_i)] - \sum_{k \in R_n, k \neq n} [\mu_k - \kappa_k][-B_{nk} - G_{nk} \times (\theta_n - \theta_k)]$$

$$- \sum_{k \in R_n} \lambda_{nk}[+B_{nk} + G_{nk} \times (\theta_n - \theta_k)] - \sum_{k \in R_n} \lambda_{kn} \times [-B_{kn} - G_{kn}(\theta_k - \theta_n)]\Big]|_{\theta_N^*, \lambda_{N\times N}^*, \mu_N^*} = 0 \tag{21}$$

$$\lambda_{kn}(L_{kn} - I_{kn})|_{\theta_N^*, \lambda_{N\times N}^*, \mu_N^*} = 0, \quad \forall k, n \in N \tag{22}$$

$$\mu_n(x_n - e_n)|_{\theta_N^*, \lambda_{N\times N}^*, \mu_N^*} = 0, \qquad \forall n \in N \tag{23}$$

$$\kappa_n(e_n + w_n)|_{\theta_N^*, \lambda_{N\times N}^*, \mu_N^*} = 0, \qquad \forall n \in N \tag{24}$$

$$\lambda_{nk} \geq 0, \kappa_n \geq 0, \mu_n \geq 0|_{\theta_N^*, \lambda_{N\times N}^*, \mu_N^*}. \tag{25}$$

2) Problem Best Response I and its KKT Conditions: In the game induced by this mechanism, user n's best response to the message m_{-n} of all other users in R_n is determined by the solution of the following problem, that we call Best Response I,

$$\min_{\hat{\theta}^n_{R_n}, p^n_{R_n}} u_n\big(e_n(\theta_{R_n}(m_{-n}, \hat{\theta}^n_{R_n}))\big) - t_n(m_{-n}, \hat{\theta}^n_{R_n}, p^n_{R_n}) \tag{Best Response I}$$

$$s.t. \quad -w_n \leq e_n \leq x_n \tag{26}$$

$$I_{nk} \leq L_{nk} \quad \forall k \in R_n \tag{27}$$

$$I_{kn} \leq L_{kn} \quad \forall k \in R_n \tag{28}$$

$$p^n_k \geq 0 \quad k \in R_n \tag{29}$$

where I_{nk} and e_n are determined by Equations (5) and (6), respectively.

This is a strictly convex optimization problem in m_n. We present the Lagrangian and the KKT conditions for the problem. The Lagrangian is

$$H_n = u_n(e_n(\theta_{R_n}(m_{-n}, \hat{\theta}^n_{R_n}))) - t_n(m_{-n}, \hat{\theta}^n_{R_n}, p^n_{R_n}) + \hat{\mu}_n(x_n - e_n) + \sum_{k \in R_n} [\hat{\lambda}_{nk}(L_{nk} - I_{nk}) + \hat{\lambda}_{kn}(L_{kn} - I_{kn})] + \sum_{k \in R_n} \hat{\gamma}_{nk} p^n_k + \hat{\kappa}_n(w_n + e_n) \tag{30}$$

Considering that

$$\frac{\partial \theta_n}{\partial \hat{\theta}^k_n} = \frac{1}{|R_n|}, \tag{31}$$

the KKT conditions at $m_{-n}, m_n^*, \hat{\lambda}^*_{nk}, \hat{\mu}^*_n, \hat{\kappa}^*_n, \hat{\gamma}^*_n$ are:

$$\frac{\partial H_n}{\partial \hat{\theta}^n_n} = u_n'(e_n) \times \frac{\partial e_n}{\partial \theta_n} \times \frac{\partial \theta_n}{\partial \hat{\theta}^n_n} - \frac{\partial t_n}{\partial \hat{\theta}^n_n} - (\hat{\mu}_n - \hat{\kappa}_n) \frac{\partial e_n}{\partial \theta_n} \frac{\partial \theta^n_n}{\partial \hat{\theta}^n_n} - \sum_{k \in R_n} [\hat{\lambda}_{nk} \frac{\partial I_{nk}}{\partial \theta_n} \frac{\partial \theta_n}{\partial \hat{\theta}^n_n} + \hat{\lambda}_{kn} \frac{\partial I_{kn}}{\partial \theta_n} \frac{\partial \theta_n}{\partial \hat{\theta}^n_n}]$$

$$= u_n'(e_n)[\sum_{i \in R_n} B_{ni} + G_{ni} \times (\theta_n - \theta_i)] \times \frac{1}{|R_n|} - l_{nn} \frac{1}{|R_n|} - 2p^n_n(\hat{\theta}^n_n - \hat{\theta}^{y_{n,n}+1}_n) + - (\hat{\mu}_n - \hat{\kappa}_n)[\sum_{i \in R_n} B_{ni} + G_{ni} \times (\theta_n - \theta_i)] \frac{1}{|R_n|} - \sum_{k \in R_n} [\hat{\lambda}_{nk}(B_{nk} + G_{nk}(\theta_n - \theta_k)) \frac{1}{|R_n|} +$$

$$\hat{\lambda}_{kn}(-B_{kn} - G_{kn}(\theta_k - \theta_n)) \frac{1}{|R_n|}] = 0 \tag{32}$$

$$\frac{\partial H_n}{\partial \hat{\theta}^n_k} = u_n'(e_n) \times \frac{\partial e_n}{\partial \theta_k} \times \frac{\partial \theta_k}{\partial \hat{\theta}^n_k} - \frac{\partial t_n}{\partial \hat{\theta}^n_k} - (\hat{\mu}_n - \hat{\kappa}_n) \frac{\partial e_n}{\partial \theta_k} \frac{\partial \theta_k}{\partial \hat{\theta}^k_n} - \hat{\lambda}_{nk} \frac{\partial I_{nk}}{\partial \hat{\theta}^n_k} - \hat{\lambda}_{kn} \frac{\partial I_{kn}}{\partial \hat{\theta}^n_k} =$$

$$u_n'(e_n)[-B_{nk} - G_{nk} \times (\theta_n - \theta_k)] \times \frac{1}{|R_k|} - l_{nk} \frac{1}{|R_k|} - 2p^n_k(\hat{\theta}^n_k - \hat{\theta}^{y_{n,k}+1}_k) + - (\hat{\mu}_n - \hat{\kappa}_n)[-B_{nk} - G_{nk} \times (\theta_n - \theta_k)] \frac{1}{|R_k|} - \hat{\lambda}_{nk}(B_{nk} + G_{nk}(\theta_n - \theta_k)) \frac{1}{|R_k|} - \hat{\lambda}_{kn}(-B_{kn} - G_{kn}(\theta_k - \theta_n)) \frac{1}{|R_k|} = 0, \quad \forall k \in R_n, k \neq n \tag{33}$$

$$\frac{\partial H_n}{\partial p_k^n} = -\frac{\partial t_n}{\partial p_k^n} = (\hat{\theta}_k^n - \hat{\theta}_k^{y_{n,1}+1})^2$$

$$+ \hat{\gamma}_{nk} = 0 \quad \forall k \in R_n \tag{34}$$

$$\hat{\lambda}_{nk}(L_{nk} - I_{nk}) = 0 \tag{35}$$

$$\hat{\mu}_n(x_n - e_n) = 0, \quad \hat{\gamma}_{nk} p_k^n \geq 0 \tag{36}$$

$$\hat{\kappa}_n(w_n + e_n) = 0, \quad \hat{\gamma}_{nk} p_k^n \geq 0 \tag{37}$$

$$\hat{\lambda}_{nk}, \hat{\kappa}_n, \hat{\mu}_n, \hat{\gamma}_n \geq 0 \tag{38}$$

3) properties of the Proposed Mechanism: We prove that our mechanism is efficient through the following results.

Theorem IV.1. *The set of NE of the game induced by the mechanism is non-empty. Furthermore, the outcome corresponding to each Nash equilibrium (NE) is a solution of the centralized problem of Section II-C (Socially Optimal I), and at each NE the mechanism is budget-balanced, individually rational and price efficient.*

Theorem IV.2. *At equilibrium, each user's payment consists of the followings: (i) his production/consumption at a price equal to his marginal utility, (ii) half of the loss for his node at a price equal to the marginal utility, and (iii) the power flow from his node to each line going out of it minus half of the loss at the congestion cost of the link. This means the producers and consumers share the cost of loss equally.*

When there is no loss in the system (i.e. DC approximation of the flows), payments are according to the nodal pricing system.

The proof of the above theorems appears in the Appendix.

C. Comparison with the Existing Designs

We compare our mechanism with other mechanisms currently used in electricity markets. The Vickrey-Clark-Groves (VCG) mechanism implements the social cost correspondence in dominant strategies. But it is not budget balanced; within the context of electricity networks, VCG results in budget surplus, but this surplus does not provide the correct signal for investment in the network's infrastructure (VCG does not use prices, thus it does not discover the right/efficient prices). Supply function mechanisms (SFM) and Cournot mechanisms are also used in electricity markets. Both of these mechanisms are budget balanced and individually rational; the Cournot mechanism is price efficient but the SFM mechanism is not. Neither Cournot nor SFM implement the social cost correspondence in Nash equilibria.

V. Market Design for Electricity Trade over the Network: Sparse Networks

Sparsity is a property of electricity networks meaning there could be nodes without any users located at them or at any of the nodes in their immediate neighborhood. We call these nodes *isolated nodes*. We extend the model presented in Section II-A to the case of sparse networks where there are isolated nodes. To address sparsity within the context of our problem we first establish the necessary notation. We denote, as before, by R_n the set of nodes in the neighborhood of node n. We define the extended neighborhood of node n, denoted by R_n^{ext}, to be the set of nodes $k \in K$ such that there exists a path from n to k that does not include any user other than the one in node n and possibly the one at node k. Furthermore, we define $\mathcal{N}_n^{ext}$ to be the set of users that belong to R_n^{ext}. We note that R_n and R_n^{ext} include node n, and $\mathcal{N}_n^{ext}$ includes the user at node n, if there is any. Note that for sparse networks, by definition $|N| \leq |K|$ and $|\mathcal{N}_n^{ext}| \leq |R_n^{ext}|$ (because there may not be any user in some of the nodes). For technical reasons we assume $|\mathcal{N}_n^{ext}| \geq 3$.

To extend our mechanism to sparse networks we proceed as in Section IV. We first define the centralized problem for sparse networks and then present the local public good mechanism. We interpret the mechanism and establish its properties.

A. The Centralized Problem

The centralized electricity dispatch problem in this case is

$$\theta_K^* = \arg \min_{\theta_k, k \in K} \sum_{n \in N} u_n(e_n(\theta_{R_n})) \quad \text{(Socially Optimal II)}$$

$$s.t. \quad -w_n \leq e_n(\theta_{R_n}) \leq x_n \quad \forall n \in N \tag{39}$$

$$I_{nk} \leq L_{nk} \quad \forall n, k \in K \tag{40}$$

$$e_m = 0 \quad \forall m \in K, m \notin N \tag{41}$$

where I_{nk} and e_n are determined by Equations (5) and (6) respectively.

B. The Local Public Good Mechanism

To address the network sparsity issue, we extend the mechanism of Section IV in the following way.

Message Space The message space of the mechanism is $M = M_1 \times M_2 \times ... \times M_N$, where M_n is the message of agent $n \in \mathcal{N}$;

$$M_n = [0, 2\pi]^{|R_n^{ext}|} \times \mathbb{R}_+^{|R_n^{ext}|}, \tag{42}$$

where $|R_n^{ext}|$ denotes the cardinality of R_n^{ext}. A message $m_n \in M_n$ is defined by

$$m_n = (\hat{\theta}^n_{R_N^{ext}}, p^n_{R_n^{ext}}), \tag{43}$$

where

$$\hat{\theta}^{\mathcal{N}}_{R_n^{ext}} := (\hat{\theta}_1^n, \hat{\theta}_2^n, ..., \hat{\theta}^n_{|R_n^{ext}|}) \tag{44}$$

$$p^n_{R^n_{ext}} := (p_1^n, p_2^n, ..., p^n_{|R_n^{ext}|}) \tag{45}$$

$1, 2, 3, ..., |R_n^{ext}|$ is an (arbitrary) indexing of the users in R_n^{ext} (this indexing is done for all sets $|R_k^{ext}|$, $k \in \mathcal{N}$), and $\hat{\theta}_k^n$ (respectively, p_k^n) is the angle (respectively, the price per unit of angle) proposed by the user in node n for node $k \in R_n^{ext}$. If node $n_0 \in R_n^{ext}$ then $\hat{\theta}^n_{n_0} = 0$. The messages $(m_n, n \in \mathcal{N})$ are communicated to the ISO.

Outcome Function The outcome function h is defined by

$$h : M_1 \times M_2 \times ... \times M_n \rightarrow [0, 2\pi]^N \times \mathbb{R}^N \tag{46}$$

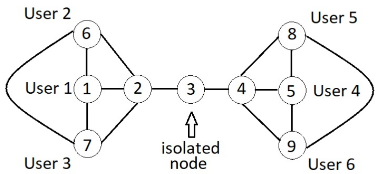

Fig. 1. Example of a network with 9 nodes, 6 users and 1 isolated node.

where for each $m := (m_1, m_2, ..., m_N) \in M_1 \times M_2 \times ... \times M_N$,

$$h(m) = (\theta_n(m_{\mathcal{N}_n^{ext}}), t_n(m_{\mathcal{N}_k^{ext}}, k \in R_n^{ext}) \tag{47}$$

$$\theta_n(m_{\mathcal{N}_n^{ext}}) = \frac{1}{|\mathcal{N}_n^{ext}|} \sum_{k \in \mathcal{N}_n^{ext}} \hat{\theta}_n^k \tag{48}$$

$$t_n((m_{\mathcal{N}^e x_k t}, k \in R_n^{ext}) = \sum_{k \in R_n^{ext}} l_{nk}((m_{\mathcal{N}_k^{ext}}), k \in \mathcal{N}_n^{ext})$$

$$\times \theta_k(m_{\mathcal{N}_k^{ext}}) + \sum_{k \in R_n^{ext}} p_k^n(\hat{\theta}_k^n - \hat{\theta}^{y_{n,k}^{ext}+1}), \tag{49}$$

$$l_{n,k}((m_{\mathcal{N}_k^{ext}}), k \in \mathcal{N}_n^{ext}) = (p^{y_{n,k}^{ext}+1} - p^{y_{n,k}^{ext}+2}) \tag{50}$$

and $y_{n,k}^{ext}$ refers to the index of user n in $\mathcal{N}_k^{ext}$ ($y_{n,k}^{ext} \neq 0$ whenever $n \in \mathcal{N}_k^{ext}, y_{n,k} = 0, n \notin \mathcal{N}_k^{ext}$); if $y_{n,k}^{ext} = |\mathcal{N}_k^{ext}|$ then $y_{n,k}^{ext} + 1 = 1$, $y_{n,k}^{ext} + 2 = 2$.

C. Interpretation of the Mechanism

The mechanism that was presented in Section IV cannot be used for sparse networks, because according to it no user bids for the angle of an isolated node; therefore, the mechanism of Section IV fails to determine the angles at isolated nodes. Moreover, the messages sent by users on the two sides of an isolated node and the outcomes for these users become completely decoupled/independent. To clarify the above statements consider the network of Figure 1 with node 1 being the slack bus/node. According to the mechanism of Section IV, users 1, 2 and 3 bid for the angles at nodes 1, 2, 6 and 7. Users 4, 5 and 6 bid for the angles of nodes 4, 5, 8 and 9. Therefore, the mechanism fails to determine the angle at node 3 that is an isolated node. Moreover, the messages sent by users 1, 2 and 3, and the outcomes for nodes $1, 2, 6$ and 7 become decoupled from the messages sent by users 4, 5 and 6, and the outcomes for nodes $4, 5, 8$ and 9.

The differences between the mechanism in this section and the one in Section IV are: (i) Each user bids for the angles and prices of all the nodes in R_n^{ext} instead of R_n. Thus, the mechanism determines the angles at all nodes of the network. (ii) The outcome angles are determined by averaging over $\mathcal{N}_n^{ext}$ instead of R_n. (iii) The payment of the user at node n is sum of all his payments for the angles of the nodes in R_n^{ext}.

D. Properties of the Mechanism

The mechanism proposed in Section V-B possesses the same properties as the mechanism proposed in Section IV. These properties can be proved in the same way as the properties of the mechanism of Section IV. We refer the reader to [44] for all the details of the proofs.

VI. Conclusion and Future Directions

In this paper, we considered electricity wholesale markets with multiple strategic users that possess localized information about the electricity network. Using ideas from auctions and local public goods, we designed a mechanism that maximizes the social welfare (the sum of the users' utilities) and has the following additional features. It satisfies the users' informational constraints along with the constraints imposed by the lines' thermal limit and the network's physical laws (Kirchhoff's laws), it is budget balanced, individually rational, and price efficient.

The model we used in our paper is a second-order approximation of the AC model. The performance of this approximation was numerically evaluated and compared to the performance of the AC model in [45], Appendix A, Table A.1. The model used in [45] is the same as the model of our paper, but in [45] it is assumed that the network users (consumers and producers) are non-strategic. Nevertheless, our market design and that of [45] (the two designs are distinctly different) result in the same power flows and angles as the solution of the centralized optimization problem for the second-order approximation model (Socially Optimal I). For this reason, the numerical results of [45] apply to/are valid for the problem studied in this paper. These results show that along individual links, the difference in flows and phase angles between the second-order approximation model and the AC model is very small.

The game form presented in this paper ensures that the desired allocations are achieved at equilibria without specifying how an equilibrium is reached. That is, this game form/mechanism does not specify an iterative process that determines how the NE of the game induced by the mechanism are computed by the producers. Determining such an iterative process is an open problem. Other open problems arise by extending/generalizing the model of Section IV. Possible generalization of the model include: (1) The case where multiple producers are located at the same node. Such a node is a *dense node* compared to the model of Section IV. As a result, even if the system operator determines the outflow of electricity power from that node, each user's share in that node is not determined yet. Determining each user's share requires local markets among the users at a dense node. (2) The case where there are M utility companies who own the N producers, and one company can own more than one producer. When multiple producers at different nodes are owned by a single generation company, this company maximizes its aggregate utility from all the producers it owns. As a result, it may strategically produce more at one of its producers (e.g. have negative utility for that single producer), in order to congest the lines and decompose/break the network into separate local markets. He can then exercise its local market power through its producers at those markets.

References

[1] A. G. Kagiannas, D. T. Askounis, and J. Psarras, "Power generation planning: a survey from monopoly to competition," *International journal of electrical power & energy systems*, vol. 26, no. 6, pp. 413–421, 2004.

[2] R. Wilson, "Architecture of power markets," *Econometrica*, vol. 70, no. 4, pp. 1299–1340, 2002.

[3] R. Sioshansi, S. Oren, and R. O'Neill, "Three-part auctions versus self-commitment in day-ahead electricity markets," *Utilities Policy*, vol. 18, no. 4, pp. 165–173, 2010.

[4] C. D. of Market Analysis, "Report on market issues and performance," in *www.caiso.com*.

[5] A. Sheffrin, "Empirical evidence of strategic bidding in the california iso real-time market," in *Electricity Pricing in Transition*. Springer, Boston, MA, 2002, pp. 267–281.

[6] F. Rahimi and A. Sheffrin, "Effective market monitoring in deregulated electricity markets," *IEEE Transactions on Power systems*, vol. 18, no. 2, pp. 486–493, 2003.

[7] J. Bushnell, B. F. Hobbs, and F. A. Wolak, "When it comes to demand response, is ferc its own worst enemy?" *The Electricity Journal*, vol. 22, no. 8, pp. 9–18, 2009.

[8] D. S. Kirschen, "Demand-side view of electricity markets," *IEEE Transactions on power systems*, vol. 18, no. 2, pp. 520–527, 2003.

[9] S. Borenstein, "The trouble with electricity markets: understanding california's restructuring disaster," *Journal of economic perspectives*, vol. 16, no. 1, pp. 191–211, 2002.

[10] S. Borenstein, J. B. Bushnell, and F. A. Wolak, "Measuring market inefficiencies in california's restructured wholesale electricity market," *American Economic Review*, vol. 92, no. 5, pp. 1376–1405, 2002.

[11] A. Ehrenmann and K. Neuhoff, "A comparison of electricity market designs in networks," *Operations research*, vol. 57, no. 2, pp. 274–286, 2009.

[12] A. Downward, G. Zakeri, and A. Philpottt, "On cournot equilibria in electricity transmission networks," *Operations research*, vol. 58, no. 4-part-2, pp. 1194–1209, 2010.

[13] W. Elmaghraby and S. S. Oren, "The efficiency of multi-unit electricity auctions," *The Energy Journal*, pp. 89–116, 1999.

[14] E. J. Anderson and X. Hu, "Finding supply function equilibria with asymmetric firms," *Operations Research*, vol. 56, no. 3, pp. 697–711, 2008.

[15] P. D. Klemperer and M. A. Meyer, "Supply function equilibria in oligopoly under uncertainty," *Econometrica: Journal of the Econometric Society*, pp. 1243–1277, 1989.

[16] R. J. Green and D. M. Newbery, "Competition in the british electricity spot market," *Journal of political economy*, pp. 929–953, 1992.

[17] R. Baldick and W. Hogan, "Polynomial approximations and supply function equilibrium stability," *Center for Business and Government, Harvard University, Cambridge, Massachusetts*, vol. 2138, 2004.

[18] R. Baldick, W. W. Hogan *et al.*, *Capacity constrained supply function equilibrium models of electricity markets: stability, non-decreasing constraints, and function space iterations*. University of California Energy Institute, 2001.

[19] R. Baldick and W. W. Hogan, "Stability of supply function equilibria implications for daily versus hourly bids in a poolco market," *Journal of Regulatory Economics*, vol. 30, no. 2, pp. 119–139, 2006.

[20] T. S. Genc and S. S. Reynolds, "Supply function equilibria with capacity constraints and pivotal suppliers," *International Journal of Industrial Organization*, vol. 29, no. 4, pp. 432–442, 2011.

[21] R. Wilson, "Auctions of shares," *The Quarterly Journal of Economics*, pp. 675–689, 1979.

[22] P. Holmberg and F. Wolak, "Electricity markets: Designing auctions where suppliers have uncertain costs," 2015.

[23] E. J. Anderson, "On the existence of supply function equilibria," *Mathematical Programming*, vol. 140, no. 2, pp. 323–349, 2013.

[24] P. Holmberg and D. Newbery, "The supply function equilibrium and its policy implications for wholesale electricity auctions," *Utilities Policy*, vol. 18, no. 4, pp. 209–226, 2010.

[25] T. Kristiansen, "Markets for financial transmission rights," *Energy Studies Review*, vol. 13, no. 1, 2004.

[26] R. E. Bohn, M. C. Caramanis, and F. C. Schweppe, "Optimal pricing in electrical networks over space and time," *The Rand Journal of Economics*, pp. 360–376, 1984.

[27] J. B. Cardell, C. C. Hitt, and W. W. Hogan, "Market power and strategic interaction in electricity networks," *Resource and Energy Economics*, vol. 19, no. 1, pp. 109–137, 1997.

[28] H.-P. Chao and S. Peck, "A market mechanism for electric power transmission," *Journal of regulatory economics*, vol. 10, no. 1, pp. 25–59, 1996.

[29] R. Wilson, "Supply function equilibrium in a constrained transmission system," *Operations research*, vol. 56, no. 2, pp. 369–382, 2008.

[30] P. L. Joskow and J. Tirole, "Transmission rights and market power on electric power networks," *The Rand Journal of Economics*, pp. 450–487, 2000.

[31] C. M. Tiebout, "A pure theory of local expenditures," *Journal of political economy*, vol. 64, no. 5, pp. 416–424, 1956.

[32] Y. Bramoullé and R. Kranton, "Public goods in networks," *Journal of Economic Theory*, vol. 135, no. 1, pp. 478–494, 2007.

[33] M. Elliott and B. Golub, "A network approach to public goods," in *Proceedings of the fourteenth ACM conference on Electronic commerce*. ACM, 2013, pp. 377–378.

[34] T. Groves and J. Ledyard, "Optimal allocation of public goods: A solution to the" free rider" problem," *Econometrica: Journal of the Econometric Society*, pp. 783–809, 1977.

[35] L. Hurwicz, "Outcome functions yielding walrasian and lindahl allocations at nash equilibrium points," *The Review of Economic Studies*, vol. 46, no. 2, pp. 217–225, 1979.

[36] Y. Chen, "A family of supermodular nash mechanisms implementing lindahl allocations," *Economic Theory*, vol. 19, no. 4, pp. 773–790, 2002.

[37] S. Sharma and D. Teneketzis, "A game-theoretic approach to decentralized optimal power allocation for cellular networks," *Telecommunication systems*, vol. 47, no. 1-2, pp. 65–80, 2011.

[38] ——, "Local public good provisioning in networks: A nash implementation mechanism," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 11, pp. 2105–2116, 2012.

[39] B. Allaz and J.-L. Vila, "Cournot competition, forward markets and efficiency," *Journal of Economic theory*, vol. 59, no. 1, pp. 1–16, 1993.

[40] J. J. Wang and J. F. Zender, "Auctioning divisible goods," *Economic Theory*, vol. 19, no. 4, pp. 673–705, 2002.

[41] J. Nash, "Non-cooperative games," *Annals of mathematics*, pp. 286–295, 1951.

[42] S. Reichelstein and S. Reiter, "Game forms with minimal message spaces," *Econometrica: Journal of the Econometric Society*, pp. 661–692, 1988.

[43] R. T. Rockafellar, *Convex analysis*. Princeton university press, Princeton, NJ, 2015.

[44] M. Rasouli and D. Teneketzis, "An efficient market design for electricity networks with strategic users possessing local information," 2019. [Online]. Available: https://umich.edu/~rasouli/ElecNetworkMarket

[45] E. Miehling, "Reasoning under uncertainty in cyber-physical systems: Toward efficient and secure operation," 2018, phD Thesis. [Online]. Available: https://bit.ly/2HA3poj

Appendix

Proof of Theorem IV.1 Let m^* be a Nash Equilibrium of the game induced by the mechanism. We establish the assertion of this theorem in the following six steps.

Step 1: We first prove that at equilibrium,

$$t_n^* = \sum_{k \in R_N} l_{nk}^* \theta_k^* \tag{51}$$

or equivalently,

$$p_k^{n*}(\hat{\theta}_k^{n^*} - \hat{\theta}_k^{y_{n,k}+1*})^2 = 0 \quad \forall k \in R_n \tag{52}$$

We establish Eq. (52) by contradiction. Assume $p_k^n(\hat{\theta}_k^n - \hat{\theta}_k^{y_{n,k}+1})^2 \neq 0$; then user n can change his price proposal to $p_k^n = 0$, without changing the rest of his message, and increase his utility. This means m_n^* is not his best response to m_{-n}^*, a contradiction.

Step 2: By construction $\sum_{k\in R_n} l_{nk}^* = 0$. Therefore, using the first step, the total payments in the system add up to zero:

$$\sum_{n\in N} t_n^* = \sum_{n\in N} (\sum_{k\in R_n} l_{nk}) \times \theta_n = 0. \tag{53}$$

This means the mechanism is budget balanced at equilibrium.

Step 3: To prove that the outcome corresponding to every NE of the game induced by the mechanism is socially optimal, it is sufficient to show that we can recover the KKT conditions of Problem (Socially Optimal I), from the KKT conditions of Problem (Best Response I). Let $\hat{\theta}_{R_n}^{n^*}, {p_{R_n}^n}^*, \hat{\lambda}_{nk}^*, \hat{\mu}_n^*, \hat{\gamma}_{nk}^*, n, \hat{\kappa}_n^*, k \in N$ describe a NE of the game along with the corresponding dual variables. We claim that the following set of variables is a solution for Problem (Socially Optimal I).

$$\theta_n = \frac{\sum_{k\in R_n} \hat{\theta}_n^{k^*}}{|R_n|} \tag{54}$$

$$\lambda_{nk} = 2 \times \hat{\lambda}_{nk}^*, n \neq k \tag{55}$$

$$\mu_n = \hat{\mu}_n^*, \kappa_n = \hat{\kappa}_n^* \tag{56}$$

For that matter we note that by Eq. (32) and (33)

$$\begin{aligned}
&\sum_{k\in R_n} \frac{\partial H_k}{\partial \hat{\theta}_n^k} = [u_n'(e_n)\frac{\partial e_n(\theta_{R_n})}{\partial \theta_n} - \hat{\mu}_n \frac{\partial e_n}{\partial \theta_n} \\
&+ \sum_{k\in R_n, k\neq n} -\hat{\lambda}_{nk}\frac{\partial I_{nk}}{\partial \theta_n} - \hat{\lambda}_{kn}\frac{\partial I_{kn}}{\partial \theta_n}] \times \frac{1}{|R_n|} \\
&+ \sum_{k\in R_n, k\neq n} [u_k'(e_k)\frac{\partial e_k(\theta_{R_k})}{\partial \theta_n} + (\hat{\kappa}_n - \hat{\mu}_k)\frac{\partial e_k}{\partial \theta_n} + \\
&- \hat{\lambda}_{nk}\frac{\partial I_{nk}}{\partial \theta_n} - \hat{\lambda}_{kn}\frac{\partial I_{kn}}{\partial \theta_n}] \times \frac{1}{|R_n|} \\
&+ \sum_{k\in R_n} l_{nk}\frac{1}{|R_n|} + \sum_{k\in R_n} -2p_n^k(\hat{\theta}_n^k - \hat{\theta}_n^{y_{k,n}+1}) = 0
\end{aligned} \tag{57}$$

Considering Eq. (57) and Eq. (52), along with the fact that by construction $\sum_{k\in R_n} l_{nk} = 0$, and taking into account the variables defined in Eqs. (54)-(56) we obtain.

$$\begin{aligned}
&u_k'(e_k)\frac{\partial e_k(\theta_{R_k})}{\partial \theta_n} + (\kappa_k - \mu_k)\frac{\partial e_k}{\partial \theta_n} \\
&- \lambda_{nk}\frac{\partial I_{nk}}{\partial \theta_n} - \lambda_{kn}\frac{\partial I_{kn}}{\partial \theta_n} = 0,
\end{aligned} \tag{58}$$

which is Eq. (21), the first KKT condition of Problem (Socially Optimal I). The other KKT conditions, Eqs. (22)-(25), hold because of Eqs. (35)-(38).

Step 4: We prove the existence of NE for the game induced by the mechanism in two stages. In the first stage, we introduce a new optimization problem for each user, called surrogate optimization problem. We show that collectively, these surrogate optimization problems have the same solution as the centralized problem (Socially Optimal I). In the second state, we use the surrogate optimization problems to prove the existence of NE for the game induced by the mechanism.

First Stage: Let $(\theta_N^*, \lambda_N^*, \mu_N^*)$ denote the unique solution and corresponding dual variables of Problem (Socially Optimal I). Define the following individual prices calculated at $(\theta_N^*, \lambda_N^*, \mu_N^*)$.

$$\begin{aligned}
&\tilde{l}_{n,n}^* = u_n'(e_n) \times \frac{\partial e_n}{\partial \theta_n} + (\kappa_n - \mu_n)\frac{\partial e_n}{\partial \theta_n} \\
&- \sum_{k\in R_n} [\frac{\lambda_{nk}}{2}\frac{\partial I_{nk}}{\partial \theta_n} + \frac{\lambda_{kn}}{2}\frac{\partial I_{kn}}{\partial \theta_n}]
\end{aligned} \tag{59}$$

$$\begin{aligned}
&\tilde{l}_{n,k}^* = u_n'(e_n) \times \frac{\partial e_n}{\partial \theta_k} - (\mu_n - \kappa_n)\frac{\partial e_n}{\partial \theta_k} \\
&- \frac{\lambda_{nk}}{2}\frac{\partial I_{nk}}{\partial \theta_k} - \frac{\lambda_{kn}}{2}\frac{\partial I_{kn}}{\partial \theta_k}.
\end{aligned} \tag{60}$$

Consider the following (individual) optimization problem for n.

$$\min_{\theta_{R_n}^{sur}} \quad u_n\big(e_n(\theta_{R_n})\big) - \sum_{k\in R_n} \tilde{l}_{nk}^*\theta_k$$

(Surrogate Optimization I)

$$s.t. \quad -w_n \le e_n \le x_n \tag{61}$$

$$I_{nk} \le L_{nk} \quad \forall k \in R_n \tag{62}$$

$$I_{kn} \le L_{kn} \quad \forall k \in R_n \tag{63}$$

where I_{nk} and e_n are determined by Equations (5) and (6), respectively. This is a strictly convex optimization in $\theta_{R_n}^n$. We present the Lagrangian and the KKT conditions for the problem. The Lagrangian is

$$\begin{aligned}
&H_n^{sur} = u_n\big(e_n(\theta_{R_n}^n)\big) - \sum_{k\in R_n} \tilde{l}_{nk}^*\theta_k + \mu_n^{sur}(x_n - e_n) + \\
&\kappa_n^{sur}(w_n + e_n) + \sum_{k\in R_n} [\lambda_{nk}^{sur}(L_{nk} - I_{nk}) + \lambda_{kn}^{sur}(L_{kn} - I_{kn})]
\end{aligned} \tag{64}$$

The KKT conditions at $\theta_{R_n}^{n*}, \lambda_{nk}^{sur*}, \mu_n^{sur*}, \kappa_n^{sur*}$ are:

$$\begin{aligned}
&\frac{\partial H_n^{sur}}{\partial \theta_n^{n,sur}} = u_n'(e_n) \times \frac{\partial e_n}{\partial \theta_n^{n,sur}} - \tilde{l}_{nn}^* - (\mu_n - \kappa_n)\frac{\partial e_n}{\partial \theta_n^{n,sur}} \\
&- \sum_{k\in R_n} [\lambda_{nk}^{sur}\frac{\partial I_{nk}}{\partial \theta_n^{n,sur}} + \lambda_{kn}^{sur}\frac{\partial I_{kn}}{\partial \theta_n^{n,sur}}] = 0,
\end{aligned} \tag{65}$$

$$\begin{aligned}
&\frac{\partial H_n^{sur}}{\partial \theta_k^{sur}} = u_n'(e_n) \times \frac{\partial e_n}{\partial \theta_k^{n,sur}} - \tilde{l}_{nk}^* - (\mu_n - \kappa_n)\frac{\partial e_n}{\partial \theta_k^{n,sur}} \\
&- \lambda_{nk}^{sur}\frac{\partial I_{nk}}{\partial \theta_k^{n,sur}} - \lambda_{kn}^{sur}\frac{\partial I_{kn}}{\partial \theta_k^{n,sur}} = 0 \quad \forall k \in R_n, k \neq n
\end{aligned} \tag{66}$$

$$\lambda_{nk}^{sur}(L_{nk} - I_{nk}) = 0 \tag{67}$$

$$\mu_n^{sur}(x_n - e_n) = 0, \quad \kappa_n^{sur}(w_n + e_n) = 0, \tag{68}$$

$$\lambda_{nk}^{sur}, \kappa_n^{sur}, \mu_n^{sur}, \gamma_n^{sur} \ge 0 \tag{69}$$

Using Eqs. (59) and (60) it is straightforward to show that $(\theta_{R_n}^*, \mu_n^*, \kappa_n^*, \frac{\lambda_{nk}^*}{2}) : k \in R_n)$ from the solution of Problem (Socially Optimal I) is a solution to the surrogate optimization problem at n. Thus, collectively these surrogate optimization problems result in the solution of the centralized optimization problem (Socially Optimal I).

Second Step: We construct a NE of the game induced by the mechanism by showing that the KKT conditions of Problem (Best Response I) are satisfied. Let $r^* = (\theta^*_N, \lambda^*_{N\times N}, \mu^*_N, \kappa^*_N)$ be the solution to Problem (Socially Optimal I). From the first stage above, we know that this is also a solution for the surrogate problems.

Consider $\hat{r}^* = (\hat{\theta}^*_N, p^*_N, \hat{\mu}^*_N, \hat{\kappa}^*_N, \hat{\lambda}^*_{N\times N})$ to be a solution to the following equations:

$$\hat{\theta}^k_n = \theta^*_n, \qquad p^k_n - p_n^{y_{k,n}+1} = \tilde{l}_{kn} \tag{70}$$

$$p^k_n \geq 0, \quad \hat{\lambda}_{kn} = \lambda^*_{kn}/2, \quad \hat{\mu}_n = \mu^*_n \quad \hat{\kappa}_n = \kappa^*_n, \forall k, n \in \mathcal{N} \tag{71}$$

Note that for Eq. (70) to have a solution, we should have

$$\sum_{j\in R_k} \tilde{l}_{jk} = 0 \tag{72}$$

which is true by Eq. (15).

The KKT conditions of Problem (Surrogate Optimization I) show that at $\hat{r}^*$, the KKT conditions for Problem (Best Response I) are satisfied and therefore, $\hat{r}^*$ defines a NE and the corresponding dual variables.

Step 5: As shown in Step 3, the outcome corresponding to any NE of the game induced by the mechanism is a solution of Problem (Socially Optimal I). The prices at equilibrium are given by Eqs. (59) and (60). These are the efficient prices because they consist of the following three parts: the marginal cost of production with respect to angles, $u'_n(e_n) \times \frac{\partial e_n}{\partial \theta_k}$, the saturation price for limited production capacity $-(\mu_n - \kappa_n)\frac{\partial e_n}{\partial \theta_k}$, and the lines congestion price $-\frac{\lambda_{nk}}{2}\frac{\partial I_{nk}}{\partial \theta^n_k} - \frac{\lambda_{kn}}{2}\frac{\partial I_{kn}}{\partial \theta^n_k}$.

Step 6: To prove individual rationality, first note that at equilibrium

$$l^*_{n,k} = \frac{\partial u_n}{\partial \theta_k}|_{m^*}. \tag{73}$$

Therefore, the total utility of user n at equilibrium is

$$u_n(e_n(\theta^*_{R_n})) - \sum_{k\in R_n} \frac{\partial u_n}{\partial \theta_k}|_{m^*} \times \theta^*_k \tag{74}$$

Since $u_n(e_n(\theta_{R_n}))$ is strictly convex in θ_{R_n} and $u_n(0) = 0$, the value of (74) at any NE m^*, which determines the cost of user n at m^*, is non-positive i.e. user n's benefit at m^* is non-negative.

Proof of Theorem IV.2 By Theorem IV.1, at the equilibrium of the game induced by the mechanism $\hat{\theta}^*_{R_n} = \theta^*_{R_n}$ and the individual prices l^*_{nk} are determined by Eqs. (59) and (60).

Define $loss_{nk}$ to be the loss in line from n to k, and $loss_n$ to be the total loss in all the lines going out of node n; then,

$$loss_{nk} = \frac{1}{2}G_{nk}(\theta_n - \theta_k)^2 \tag{75}$$

and

$$loss_n = \sum_{k\in R_N} \frac{1}{2}G_{nk}(\theta_n - \theta_k)^2 \tag{76}$$

The equilibrium payment to the user located at node n, according to Eqs. (53), (59) and (60), is

$$t^*_n = \sum_{j\in R_n} l^*_{nj}\theta^*_j = [u'_n(e^*_n) - \mu^*_n + \kappa^*_n] \tag{77}$$

$$\times [\sum_{j\in R_n, j\neq n} (-B_{nj} - G_{nj}(\theta^*_n - \theta^*_j)) \times \theta^*_j$$

$$+ (\sum_{j\in R_n} B_{nj} + G_{nj} \times (\theta^*_n - \theta^*_j)) \times \theta^*_n]$$

$$+ \sum_{j\in R_n} \frac{\lambda^*_{nk} + \lambda^*_{kn}}{2}[B_{nk}(\theta^*_n - \theta^*_k) + G_{nk}(\theta^*_n - \theta^*_k)^2] \tag{78}$$

$$= [u'_n(e^*_n) - \mu^*_n + \kappa^*_n] \times [e^*_n + \frac{1}{2}Loss^*_n] +$$

$$\sum_{j\in R_n} (\frac{\lambda^*_{nk} + \lambda^*_{kn}}{2})(I_{nk} + \frac{loss^*_{nk}}{2}) \tag{79}$$

This payment consists of the components mentioned in the statement of the Theorem.

When the loss is zero, the payments are equal to those of the nodal pricing system.

Mohammad Rasouli received his Ph.D. in Electrical Engineering: Systems in 2017 from the University of Michigan where he also received a M.Sc. in economics in 2014. Prior to joining the University of Michigan, he attended Sharif University of Technology where he received the B.Sc. in Electrical Engineering in 2009 and the M.Sc. in Electrical Engineering: Communication Systems in 2011. His research interests include mechanism design, network economics, and stochastic control with applications in energy systems.

Demosthenis Teneketzis (M'87–SM'97–F'00) received his Ph.D. degree in electrical engineering from the Massachusetts Institute of Technology, Cambridge, MA, USA. He is currently Professor of Electrical Engineering and Computer Science at the University of Michigan, Ann Arbor, MI, USA. His research interests are in stochastic control, decentralized systems, queuing and communication networks, stochastic scheduling and resource allocation problems, mathematical economics, and discrete-event systems.